The problem of constructing data-flow information from the control-flow graph of a program has been studied by a number of investigators [1,2,3,4,5,6]. In [3] the author presented an algorithm which uses "Cocke-Allen interval" analysis to solve the problem of locating "live" variables in a program. Hecht and Ullman [4] proposed a tabular method which has been shown to require more bit-vector operations than the interval method on some graphs and fewer on others [5].

This newsletter proposes an entirely new tabular approach which is applicable to most global data-flow problems. After an initial processing expenditure, this method is optimal in terms of bit-vector operations.

**Edge-Listings**

We define an *edge-listing* to be a sequence

\[ \ell = (e_1, e_2, \ldots, e_m) \]

of edges in the program flow graph, where some edges may be repeated, such that every simple path in the flow graph is a subsequence of \( \ell \). That is, if

\[ (d_1, d_2, \ldots, d_n) \]

are the edges of a simple path in the flow graph, there exist indices

\[ j_1, j_2, \ldots, j_n \]

such that \( j_i < j_{i+1}, \ 1 \leq i < n \), and \( d_i = e_{j_i}, \ 1 \leq i \leq n \).
Certainly an edge-listing exists, because if \( f_1, \ldots, f_k \) are all the edges in the graph then

\[
\lambda = (f_1, \ldots, f_k, f_1, \ldots, f_k, \ldots, f_1, \ldots, f_k)
\]

with \( k \) repetitions of \((f_1, \ldots, f_k)\) is a valid edge-listing. An edge-listing is said to be \textit{minimal} if there is no shorter edge-listing for the same graph.

\textbf{Data Flow}

Most data flow problems can be expressed by "edge-equations" on the edges of the control flow graph. For example, consider the problem of identifying "live" variables. The following sets are important.

1. \textit{live}(b) - the set of variables which are live on entry to the block \( b \) - where a variable is "live" at a point if there exists a path from that point to a use of the variable which path contains no redefinition of the variable.

2. \textit{inside}(b) - the set of variables for which there is a use not preceded by a definition in \( b \).

3. \textit{thru}(b) - the set of variables which are neither used nor defined in \( b \).

It has been shown \([3,5]\) that the following class of equations defines the problem:

\[
(1) \quad \text{live}(b) = \text{inside}(b) \cup \bigcup_{k \in S(b)} (\text{thru}(b) \cap \text{live}(k))
\]
where \( S(b) \) is the set of successors of the block \( b \) in the control flow graph.

The essence of the edge-listing method is to propagate the "live" information backwards along all simple paths by applying the following analog of equation (1)

\[
(2) \quad \text{live} (b) = \text{live}(b) \cup (\text{thru}(b) \cap \text{live}(k))
\]

\( (\text{live}(b) \) is initially \( \text{inside}(b) \)\) on each edge \((b,k)\) of an edge-listing in reverse order. The following SETL algorithm does this. Its argument \( \text{edgelist} \) is an edge-listing represented as a tuple of pairs \( <b,k> \).

```setl
define liveanalysis (nodes, edgelist, thru, inside)
/* initialize live to inside */
(V b £ nodes) live(b) = inside(b); end V b;
/* iterate through the edge-listing */
(# edgelist £ V i > 1 ) /* reverse order */
<b,k> = edgelist(i);
live(b) = live(b) + (thru(b) * live(k));
end V i;
return live;
end liveanalysis;
```

This simple algorithm is optimal whenever the edge listing is minimal.

Our only remaining problem is to find an algorithm which generates a minimal edge listing.

**Enumerating Simple Paths**

The first step in the generation of a minimal edge listing is the enumeration of all simple paths in the graph. For each node \( n \), we will compute \( path(n) \), the set of simple paths which terminate at \( n \). This computation can be performed by an iterative method similar to Kildall's [6].
Initially, $\text{paths}(n)$ contains only the null tuple. We iterate through the nodes $n$ of the graph adding to $\text{paths}(n)$ all paths leading to a predecessor $k$ of $n$ and through $k$ to $n$ (which paths do not contain $n$). When no new paths are added during an iteration, we halt.

The following is a SETL function which accepts $\text{graph}$, a tuple of nodes, edges, and entry node, and produces the paths sets.

define computepaths (graph);
    /* break graph into component parts */
    <nodes, edges, entry> = graph;
    /* compute the predecessor function */
    preds = \{<tlx, hd x> x \in edges\};
    /* initialize the paths to null tuples */
    (\forall n \in nodes) paths(n) = \{null\}; end \forall n;
    /* iterate through the graph until there are no changes */
    change = t;
    (while change) . change = f;
        /* recompute paths for each node in the graph */
        (\forall n \in nodes)
            newpaths = [+: k \in preds \{n\}] paths(k);
            /* check each new path for the occurrence of $n$ */
            (\forall p \in newpaths)
                if (1 \leq \forall j \leq \# p, | hd p(j) ne n)
                    and (p + \langle k,n \rangle) \in paths(n)
                    then paths(n) := paths(n) with (p + \langle k,n \rangle);
                    change = t; end if;
            end \forall p;
        end \forall n;
    end while;
    return paths;
end computepaths;
Our next task is to eliminate some duplication. We take all the paths into one set and eliminate those which are contiguos within another path.

```plaintext
define reduce (paths);
    /* pool all paths */
    allpaths = [+ : x ∈ paths] t x ;
    /* check for contiguous subsequences */
    (∀t ∈ allpaths ∃ ( q ∈ allpaths − {t}, 1 ≤ j ≤ (#q−#t+1)
    | q(j: #t) eq t ))
    allpaths = allpaths − {t};
    ∀t;
    return allpaths;
end reduce;
```

On completion of this function, we are left with a set of maximal simple paths.

**Edge-Listing Generation**

We must now find an edge sequence which contains each of the enumerated paths as a subsequence. To do this we will use an exhaustive search through a tree of possibilities.

We build a tree in which we have two quantities associated with each node:

1. `sofar(node)` - the partial edge-listing generated so far.
2. `rpaths(node)` - the set of paths remaining to be accounted for.
In other words, each node represents a sequence of decisions which have led to the partial listing \textit{sofar}(node). When a new node is created by a decision to add a certain edge to the listing, that edge is stripped off the beginning of each remaining path to create a new \textit{rpaths} set. When the set of remaining paths reduces to a null tuple, the edge-listing is complete. Each leaf in the completed tree represents a valid edge-listing, the shortest being a minimal edge-listing.

The SETL function \texttt{multimerge} uses an adaptation of this method. The tree is built in stages: first all possible initial sequences of length 1 are computed, then those of length 2, and so on. We stop whenever all paths have been exhausted for some leaf because when this happens we will have a minimal listing. A small speed-up is attained by the following trick: Whenever an edge appears only at the beginning of the remaining paths we can add this edge to the partial sequence without considering other possible decisions - thus we can move to the next stage with only one son tending from this node.

The following SETL function creates a new tree node, given the sets \textit{sofar} and \textit{rpaths} for the parent node (\texttt{seq} and \texttt{rempaths}, respectively) and the selected next edge \texttt{x}. If, after stripping \texttt{x} from the remaining paths, all paths have been accounted for, the edge listing is returned as the value; otherwise, the new node is added to the argument set \texttt{new}.

\begin{verbatim}
define createnode ( x, seq, rempaths, new);
   /* get a new atom */
   node = newat;
   sofar(node) = seq + < x >;
\end{verbatim}
/* strip initial edges from remaining paths to create a new rpaths set */

rpaths(node) = n;  
(∀t ∈ remset)
  if x ∈ t(l)
    then rpaths(node) = rpaths(node) with (t t);
    else rpaths(node) = rpaths(node) with t;
  end if x;
end ∀t;
/* test to see if remaining paths are null */
if rpaths(node) ≡ {nul}
  then return (sofar(node));
  else new = new with node;
       return nul;
  end if;
end createnode;

Finally we present the routine multimerge which builds the tree. Note that the operator seqelt returns the index of an element in a tuple.

define multimerge (pathset)
  /* initialize the first node */
  node = newat;  
rpaths(node) = pathset;
 sofar(node) = nul;
  nodes = {node};
  (while nodes ≠ n1 doing nodes = new; new = n1;)
  /* iterate through the tree nodes at this stage */
  (∀n ∈ nodes)
    <seq, remset> = <sofar(n), rpaths(n)>
    /* check for special condition */
    if 3t ∈ remset | (∀q ∈ remset | (t(l)seqelt q)≤ l)
      then /* create only one son */
test = createnode (t(1), seq, remset, new);
if test ne nult then return test;;
else /* multiple branches */
    startset = hd [remset];
    (∀x ∈ startset)
    test = createnode(x, seq, remset, new);
    if test ne nult then return test;;
    end ∀x;
end if;
end ∀n;
end while;
end multimerge;

The operator seqelt is programmed as follows.

define a seqelt t;
    return (if 1 ≤ i ≤ # t | t(i)eq a then i else 0);
end segelt;

The method we have presented is clearly "brute force". It is probably not worth the effort to compute an edge-listing with such a time-consuming method. However, it does provide us with a method of evaluating heuristic edge-listing generators.

Summary

We have introduced the concept of a minimal edge-listing and we have used such a listing to rapidly compute global data-flow information. A time-consuming algorithm to generate a minimal edge-listing has been presented. This algorithm will be used in the evaluation of heuristic generators to be discussed in future newsletters.
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References


