THE LOGLISP USER'S MANUAL

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Note

This document describes version V2M3 ("version 2, modification 3") of LOGLISP. It supersedes [Robinson-Sibert 1980], which describes version V1M1 of LOGLISP. The LOGLISP project is a continuing experiment and further versions and modifications of the system will be issued from time to time as seems appropriate. Successive versions VnMj of LOGLISP are essentially upward compatible. Occasionally, however, we have sacrificed upward compatibility in order to install obvious improvements. We believe that the user will experience little or no confusion on this account.
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This document describes version V2M3 of LUGLISP, an extension of LISP in which one can do logic programming (Kowalski 1974, 1979). It is a revision and extension of our "LOGIC PROGRAMMING IN LISP" (Robinson-Sibert 1980), which described previous versions of LUGLISP. LUGLISP is basically UCI LISP (Meehan 1979) with a logic programming system embedded within it. The LUGLISP user we have in mind is thus (ideally) someone who is familiar with UCI LISP (or, at least, LISP); we do not in this manual address the non-LISP community, beyond some general discussion in the first few chapters.

We do not assume the user is already familiar with logic programming or the earlier background involving resolution theorem-proving. The early chapters attempt to provide an overall view of the essential ideas in a fairly general setting. In particular no prior acquaintance with PROLOG is assumed. In order to distinguish our work from that of our PROLOG colleagues (which, and whom, we esteem highly) the logic programming system within LUGLISP is called LOGIC. Thus we have: LUGLISP = LOGIC + LISP. The present version of LUGLISP has been improved considerably over earlier versions, both in the efficiency of the implementation and in the incorporation of several new features which we believe will be found useful.

The reader interested in details of the implementation is referred to the companion document "LUGLISP Implementation Notes" (Robinson-Sibert 1981) which is an up-to-date version of the chapter on implementation in "LOGIC PROGRAMMING IN LISP". The corresponding chapter has been removed from the present document.

LOGIC differs in a number of ways from the well-known PROLOG implementations of logic programming (Roussel 1975), (Warren 1977), (Roberts 1977), (Clark 1979). The most noteworthy difference is that LOGIC is simply a set of new LISP primitives designed to be used freely within LISP programs. These primitives are invoked in the ordinary LISP manner by function calls from the terminal or from within other LISP programs. They return their results as LISP data objects which can be subjected to analysis and manipulation. Each of the logical procedures comprising a LOGIC knowledge base is a LISP data object kept (like the definition of an ordinary LISP procedure) on the property list of the identifier which is its name.

Thus one calls LOGIC from within LISP. It is also possible to call LISP from within LOGIC. The identifiers used as logical
predicate symbols, function symbols and individual constants within a knowledge base or query can be given a LISP meaning by the ordinary LISP method of definition or assignment. Some identifiers (CAR, CONS, PLUS, etc.) already have a LISP meaning imposed by the system. Thus every logic construct (term, or atomic sentence) is capable of being interpreted as a LISP construct. During the deduction cycle of LOGIC each logic construct is "evaluated" as a LISP construct, according to a suitably generalized notion of evaluation.

The effect of this LISP-simplification step within each deduction step is to make available to the LOGIC programmer virtually the full power of LISP. This makes trivially easy the "building-in" of "immediately evaluable" notions - but far more than that. In particular, LOGIC calls can be made from within LOGIC calls.

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Since Kowalski's 1974 paper "Predicate Logic as Programming Language" (Kowalski 1974) there has been a growing interest in the use of what he calls "logic programming" as a technique for specifying computations.

This technique consists of formulating computational specifications as a set of declarative sentences, each of which is a simple assertion of some truth - conditional or unconditional, general or particular - which one wishes to record in a "knowledge base".

A conditional assertion has the form

\[ \text{\textit{a}} \text{ if } \text{\textit{A}} \]

in which \textit{a} is the conclusion and \textit{A} is the hypothesis. The hypothesis is a list

\[(\text{\textit{A}}_1 \ldots \text{\textit{A}}_n)\]

of conditions \textit{A}i all of which (the assertion is saying) must be true in order that the conclusion be true. As a special case, the list \textit{A} may be empty. Such a hypothesis is always true, and so the assertion in this case is said to be unconditional.

Kowalski writes the conditional assertion

\[ \text{\textit{a}} \text{ if } (\text{\textit{A}}_1 \ldots \text{\textit{A}}_n) \]

as

\[ \text{\textit{a}} \leftarrow \text{\textit{A}}_1 \ldots \text{\textit{A}}_n \]

and the unconditional assertion

\[ \text{\textit{a}} \text{ if } () \]
Each of the A's and the a is a sentence in subject-predicate form, or "predication":

\[(P \; S_1 \ldots \; S_k)\]

In which some predicate \(P\) is ascribed to a subject \((S_1 \ldots \; S_k)\) which in general is a tuple of descriptive expressions each of which is either a proper name, or a variable, or an applicative expression

\[(F \; S_1 \ldots \; S_n)\]

In which some operator \(F\) is applied to some operand \((S_1 \ldots \; S_n)\). The operand of a construction is in general a tuple of descriptive expressions of just the same kind as the subject of a predication.

An assertion containing one or more variables is **general** (a *generalized*). In practice, it is found that most generalities are conditional. However, unconditional generalities are quite meaningful and occur naturally in many applications.

An assertion containing no variable is **particular** (a *particularized*). In practice, particularities are almost always unconditional. Conditional particularities are, however, meaningful and occasionally occur naturally. Particular expressions (not just predications) are also called **ground expressions**.

These remarks lead us to introduce a slightly different classification of assertions. An unconditional particular assertion is said to be a **rule**. Any other assertion is said to be a **goal**. For commonly occurring assertions, this is not different from the division into particular and general assertions, but the distinction is important in the implementation.

The variables in a general assertion are treated as if they were governed by universal quantifiers preceding the assertion. Thus, the assertion

\[(\text{Odd} \; \text{Product} \; x \; y) \; \leftarrow \; (\text{Odd} \; x) \; (\text{Even} \; \text{Sum} \; x \; y))\]
should be understood as being preceded by "for all x and y". Once a knowledge base has been built the logic programmer can request **prove** to **reductio**. It is these requests which invoke the "logic computations" or **reductio** which reveal the implicit content of the knowledge base.

A query is essentially a description

\[ \forall (x_1 \ldots x_k) \text{ such that } (A_1 \text{ and } \ldots \text{ and } A_n) \]

of a set of tuples which satisfy a given conjunction (the **constraint** of the query).

The constraint of a query may contain variables in addition to those occurring in the **answer template** \((x_1 \ldots x_k)\) of the query. These are to be understood as being governed by existential quantifiers preceding the constraint.

The answer to such a query is then the set of all tuples whose satisfaction of the given constraint follows logically from the knowledge base.

Thus the answer may be the empty set, or a set containing just one tuple, or a set containing many - even infinitely many - tuples. If the answer set is infinite, then in practice some finite subset of it will be supplied, or some other description of the set will be given.

A logic computation, then, consists of the sequence of events necessary to construct the answer to some query from the information embedded in some knowledge base.

1.9 PROLOG

These ideas were incorporated into a programming language called **PROLOG**, designed and first implemented by a group at the University of Marseille. **PROLOG** has since been implemented at the universities of Edinburgh, Leuven, London, Waterloo and Budapest.

**PROLOG** implementations of logic programming go beyond the "pure" version of it described by Kowalski. They provide certain "imperative" features by which the programmer can affect the deductive computation of the answer to a query, and indeed by which he can affect the meaning of the query and of the assertions in the knowledge base.

These "control constructs" of **PROLOG** have been found most useful
In practical applications of logic programming and we are in no sense critical of them. However, we believe that it is one of the essential ideas of logic programming to make a clean distinction between the "logic" of one's program and its "control".

2.0 LOGIC

Accordingly we have implemented a programming language called LOGIC, which embodies our idea of the "pure" version of logic programming featured in Kowalski's writings. Those who are interested to experiment with "pure" logic programming can do so by working with LOGIC.

For those who may wish to avail themselves — while still in some sense working within a logic programming framework — of a greater degree of algorithmic control over events, we have embedded LOGIC within a system called LOGLISP.

3.0 LOGLISP = LOGIC + LISP

LOGLISP is a marriage of LOGIC with LISP.

A LOGLISP workspace contains everything one expects to find in a LISP workspace, and can be used purely as such by those who wish to ignore the presence of LOGIC in that workspace.

The same LOGLISP workspace can also be used as a "pure" LOGIC workspace, that is, as nothing but a basic logic programming environment, in which the assertion/query style of computing can be conducted in just the Kowalski manner. The logic programming facilities are invoked by making suitably-formed LISP calls on such LISP functions as `assert` and the query functions `all`, `any`, `the`, and `setof`. These LISP functions, together with further auxiliary and supplementary LISP functions, comprise the LOGIC system.

A major advantage of embodying logic programming within LISP in this way is that the LISP environment is available to the logic programmer as a convenient host facility in which LISP functions for editing, displaying, monitoring, debugging, inputting and outputting one's assertions, queries and deductions can be invoked interactively or under program control.

Since the putting of a query is just the submission of an appropriate LISP function call, this can be done either (as in the PROLOG systems) interactively from the terminal or internally from within an applications program.
Since the answer to a query is a LISP data object it can either (as in PROLOG) be displayed on the terminal as a stream or returned to an internal call as its result and subjected, if desired, to analysis and manipulation.

Both predicates and operators in logic expressions can be given a LISP meaning by suitable programmer-supplied definitions of them as LISP function names. Some proper names indeed have a LISP meaning which is present in every workspace as part of LISP itself.

By design extension of the "pure" logic programming paradigm, LOGLISP is capable of recognizing such predicates and operators during the deduction cycle of LUCIC. The predications and constructions in whose heads they occur are thereby treated as LISP-meaningful function calls, and are replaced in situ by appropriate simplifications.

The effect of this LISP-simplification step, performed once in every iteration of LOGIC's deduction cycle, is to give the LOGIC programmer the means to invoke very nearly the full power of LISP from within logic expressions.

This fact, together with the previously mentioned fact that LOGIC calls are simply certain LISP calls, means that it is very easy to initiate subordinate deductions during a deduction, by making recursive calls on LOGIC from within LOGIC.

Thus LISP is not only a rich and convenient host environment for LOGIC programming, but also an intimately involved partner in the novel hybrid style of "LOGLISP" programming in which LISP and LOGIC call each other, and themselves, recursively.

The following chapters describe LOGLISP in full. The background ideas are explained in detail, and the design and implementation are presented both "top-down" and "bottom-up". Examples of applications of LOGLISP are given which illustrate its novel capabilities.

LOGLISP runs on the DEC-10 under the TOPS-10 operating system; using a version of Rutgers-UCI LISP, essentially that described in [Keenan 1979].
CHAPTER 1

NOTIONS AND NOTATIONS

In this manual we are concerned with computations whose data are expressions. It will be useful to have the basic ideas and notational conventions available from the outset, and in this chapter we discuss the most important of these. The general framework is that of LISP, augmented in certain ways to accommodate the needs of LOGIC.

1.1 EXPRESSIONS

LISP has two kinds of expression: atoms and dotted pairs. We further divide the atoms into two kinds: variables and proper names. Therefore we have three kinds of expression:

variables
proper names
dotted pairs

A variable is an identifier which begins with a lower case letter. A proper name is any atom which is not a variable (in particular strings and numerals are proper names). A dotted pair is a composite expression with two immediate constituents, called its head and its tail, both of which are expressions. We have three formal predicates, for use in writing algorithms, which correspond to the three kinds of expression.

(VAR A) = TRUE if A is a variable, = FALSE otherwise
(NAME A) = TRUE if A is a proper name, = FALSE otherwise
(CONSP A) = TRUE if A is a dotted pair, = FALSE otherwise

1.2 NOTATION FOR DOTTED PAIRS AND LISTS

When A is a dotted pair whose head is B and whose tail is C, we write

\[ \textbf{\ldots} = \text{hA} \]
\[ \textbf{\ldots} = \text{tA} \]

using the decomposition functions \( \text{h} \) and \( \text{t} \). To indicate the composition of \( A \) from \( B \) and \( C \) we write:

\[ -1-1- \]
A = (B,C)

using the composition function . ("dot") written between its two arguments. In writing nested compositions with the infixed dot we may omit pairs of parentheses with the understanding that association is to the right. Thus

A•B•C•D•E•F•G

is short for

(A•(A•(C•(D•(E•(F•G)))))

A further notational economy is achieved by identifying certain expressions as lists and writing them without dots. All lists are dotted pairs except for one, which is the proper name: NIL. NIL is known as the empty list, and may also be written: (). Lists other than () are said to be nonempty. A nonempty list is any dotted pair whose tail is a list. A nonempty list may be written by writing its one or more components in order, with a left parenthesis before the first component and a right parenthesis after the last. The head of a list is its first component, and in general the (i + 1)st component of a list is the i th component of its tail. Thus the list

(D•(2•(4•(6•(BINGO•NIL))))

has six components and would be written

(D 2 4 6 BINGO)

Note that the tail of a nonempty list is just the list of its remaining components after the head has been removed.

Both list- and dot-notations are blended together in the convention whereby, e.g., A•B•C•D•E•F•G can be written

(A B C D E F G)

showing that it is like a nonempty list in having successive components, but unlike a list in that its "final tail" is not NIL. In general, an arbitrary right-associated nest of dotted pairs

([x1 x2 ... (xn xn+1) ...])

is writable as the "dotted list"
(x1 x2 ... xn * xn+1)
and as the list
(x1 x2 ... xn)
in the special case that xn+1 is NIL.

Certain formal notions are used for computing with lists. The result of concatenating two lists L and M is written L&M and is defined by

L&M = if L is () then M else (hL).((tL)*M)

Thus

(1 2 3)*(4 5 6) = (1 2 3 4 5 6)

The length of a list is the number of components it has:

(LENGTH L) = if L is () then 0 else 1 + (LENGTH tL)

1.3 PATHS. STRUCTURES. PRINTABLE EXPRESSIONS

The notion of a path is helpful in understanding the structure of expressions.

The decomposition functions h and t are the two paths of length 1. The functions hh, ht, th, tt are the four paths of length 2. In general the 2^(n+1) paths of length n+1 are all the functions h^p, t^p where p is a path of length n. The identity function I is the (only) path of length 0. An expression is said to admit a path p if the result of applying p to it is defined. Thus, every expression admits I, and every dotted pair also admits h and t. Variables and proper names admit only I, and this fact is their characteristic structural property. In general the set of all paths admitted by an expression A is called the structure of A, and gives a rather direct portrayal of A's "shape".

A useful way to represent an expression is as a connected directed graph with two kinds of node - atoms and dotted pairs. A node which is an atom has no out-arcs. A node which is a dotted pair has exactly two out-arcs, one labelled "h" and the other labelled "t". Each arc impinges upon exactly one node. Each node which is an atom is labelled by its "printname". There is a distinguished node called the root of the expression, from which there is at least one chain of arcs to every node in the expression. Each such chain beginning at the root node describes...
in the obvious way a composition of the functions $h$ and $t$ (the one obtained by reading the labels on the successive arcs of the path in reverse order). Such a graph $G$ represents the expression $A$ if the paths admitted by $A$ are exactly those described by the chains of $G$, and if when $pA$ is an atom $X$, the chain describing $p$ in $G$ has $X$ as its terminal node.

An expression may have many - even infinitely many - such representations as a graph.

Thus the expression whose head is $0$ and whose tail is itself can be represented by the graph:

```
      h
   -------> * -------> * 0
   |      |
   |      t
   |  ----
   
```

with two nodes, one of which is a dotted pair and the root, the other of which is the atom $0$.

In such an expression-graph two chains are equivalent if they lead from the root to the same node. Thus in the above graph there are two equivalence classes of chains, namely those describing the paths in

$$(1, t, tt, ttt, \ldots)$$

and those describing the paths in

$$(h, ht, htt, httt, \ldots).$$

This illustrates how in general the paths admitted by a given expression $A$ are partitioned by each graph $G$ which represents $A$ into equivalence classes which correspond abstractly to the nodes of $G$. The class containing $I$ always corresponds to the root. In general the system of equivalence classes shows how the structure of the expression is "shared" when represented by a graph. The same expression can have different sharing systems, corresponding to the different graphs which represent it. For example, the expression whose head is $0$ and whose tail is itself can be represented by many other graphs, such as the (infinite) tree whose sharing classes are

$$(1), (h), (t), (ht), (tt), \ldots.$$
that is, all singletons. In this representation there is no sharing at all.

The printable expressions are those whose structure is finite. Not all expressions are printable. For example, the dotted pair whose head is 0 and whose tail is itself is not printable, since its structure is the infinite set of paths

\[ \{ 1, t, tt, ttt, \ldots, ht, htt, httt, \ldots \} \]

It may be described as the expression which solves the equation

\[ x = 0 \cdot x \]

and we may reason about it from this description. We may also represent it as a finite (cyclic) graph as discussed above. However, to attempt to print it would result in a nonterminating process.

1.4 ENVIRONMENTS

A dotted pair whose head is a variable is called a binding. A list whose components are bindings with distinct heads is called an environment. Intuitively an environment is a collection of replacement instructions coded as dotted pairs, each one saying that a certain variable (its head) is to be replaced by a certain expression (its tail). An environment which contains all the bindings of the environment \( E \) (and perhaps other bindings) is called an extension of \( E \).

1.5 THE NOTION DEF

If \( E \) is an environment and \( A \) is an expression we say that \( A \) is defined in \( E \) if, and only if, there is a binding in \( E \) whose head is \( A \). Accordingly we introduce the function DEF by the scheme

\[
(\text{DEF } A \; E) = \begin{cases} \text{FALSE} & \text{if } E \text{ is } () \text{ then } \text{FALSE} \\ \text{TRUE} & \text{else if } \text{head } E \text{ is } A \text{ then } \text{TRUE} \\ \text{DEF } A \; \text{tail } E & \text{else} \end{cases}
\]

which computes the truth value that \( A \) is defined in \( E \). Note that if \( A \) is defined in \( E \) then \( A \) is a variable.

1.6 THE NOTIONS IMM AND ULF

If \( A \) is defined in \( E \) we say that the immediate associate of \( A \) in \( E \) is the tail of the binding in \( E \) whose head is \( A \), and we define the corresponding function IMM by

\[ \text{IMM}(A, E) = \text{tail } (A \; E) \]
(IMM A E) = if the is A then the else (IMM A tE) with the understanding that IMM will never be invoked for an A and E such that A is not defined in E. The immediate associate in E of a variable A may itself be a variable defined in E. In such a case we may wish to track down the ultimate associate of A in E - namely the first expression in the series

A, (IMM A E), (IMM (IMM A E) E) ..., which is not defined in E. Accordingly we define the function

ULT by

ULT A E = if (DEF A E) then (ULT (IMM A E) E) else A

which computes, for any expression A and environment E, the ultimate associate of A in E. For example, if E is the environment

( (x , y , z , (F A (a , r , s )) r , (a , G s )) s , 5 )

then the immediate associate of x in E is y, but the ultimate associate of x in E is (F A (a , r , s )).

1.7 REALIZING EXPRESSIONS IN ENVIRONMENTS

Given an expression A and an environment E, we consider the result of replacing each variable in A by its immediate associate in E. This expression is called the realization of A in E. To compute the realization of A in E we use the function REAL, defined by:

(REAL A E) = if (CONSP A) then (REAL (CAR A) E) . (REAL (CDR A) E)
           else if (DEF A E) then (IMM A E)
                   else A

we note that, for example, the realization of (PLUS x y) in the environment

( (x , y , z , (F A (a , r , s )) r , (a , G s )) s , 5 )

is (PLUS y z). We are also interested in recursive realizations. For example, if we start with (PLUS x y) we obtain each of the following expressions by repeatedly realizing the previous one in E:

(PLUS y z)
(PLUS z (F A (b , r , s )))
(PLUS (F A (b , r , s )) (F A (b , (G s ) , 5 )))

- 1-6 -
Realizing the final expression in E merely reproduces it. This final expression is therefore by definition the recursive realization of \((\text{PLUS } x \ y)\) in E. In general the recursive realization of an expression \(A\) in an environment \(E\) is defined by:

\[
(\text{RECREAL } A \ E) = \begin{cases} 
\text{IF } (\text{CONSP } A) \text{ then } (\text{RECREAL } NA \ E). (\text{RECREAL } TA \ E) \\
\text{else IF } (\text{UEF } A \ E) \text{ then } (\text{RECREAL } (\text{ULT } A \ E) \ E) \\
\text{else } A
\end{cases}
\]

1.8 UNPRINTABLE RECURSIVE REALIZATIONS OF PRINTABLE EXPRESSIONS

It can happen that a printable expression may have an unprintable recursive realization in a printable environment. For example, in the environment

\[
(\ x.([0.\ x]\ ) )
\]

the expression \(x\) has the recursive realization

\[
([0.([0.\ ...])])
\]

which is the "infinite expression" whose head is 0 and whose tail is itself.

1.9 UNIFICATION

A fundamental notion in logic programming is the operation of unifying two expressions \(A\) and \(B\) relative to a given environment \(E\). This operation yields a result, denoted by \((\text{UNIFY } A \ B \ E)\), which is either the message "IMPOSSIBLE", indicating that \(A\) and \(B\) cannot be unified with respect to \(E\), or else is an extension of \(E\) in which the recursive realizations of \(A\) and \(B\) are identical. In the latter case we say that the environment \((\text{UNIFY } A \ B \ E)\) is the most general unifier ("mgu") of \(A\) and \(B\) with respect to \(E\). By definition, we then have that

\[
(\text{RECREAL } A \ (\text{UNIFY } A \ B \ E)) = (\text{RECREAL } B \ (\text{UNIFY } A \ B \ E))
\]

The computation of \((\text{UNIFY } A \ B \ E)\) is defined by

\[
(\text{UNIFY } A \ B \ E) =
\begin{cases} 
\text{"IMPOSSIBLE" } \text{if } E \text{ is "IMPOSSIBLE"} \\
\text{"IMPOSSIBLE" } \text{else if } (\text{UEF } A \ E) \text{ then } (\text{UNIFY } (\text{ULT } A \ E) \ E) \\
\text{else (EQUATE (ULT A E) (ULT B E) E)}
\end{cases}
\]
where

\[
\text{EQUATE } A \leftrightarrow E = \\
\begin{cases} 
\text{if } A \text{ is } B \text{ then } E \\
\text{else if } (\text{VAR } A) \text{ then } (A,B),E \\
\text{else if } (\text{VAR } B) \text{ then } (B,A),E \\
\text{else if not } (\text{CONS } A) \text{ then } "\text{IMPOSSIBLE}" \\
\text{else if not } (\text{CONS } B) \text{ then } "\text{IMPOSSIBLE}" \\
\text{else } (\text{UNIFY } A \leftrightarrow E) \leftrightarrow (\text{UNIFY } A \leftrightarrow E) 
\end{cases}
\]

The mu of \( (P \ (G \ x \ y) \ x \ y) \) and \( (P \ a \ (H \ b) \ c) \) with respect to the empty environment (\( \_ \)) is

\[
( y \cdot c \ x \cdot (H \ b) \ a \cdot (G \ x \ y) )
\]

and in this environment both expressions are recursively realized as

\[
(P \ (G \ (H \ b) \ c) \ (H \ b) \ c)
\]

The mu of \( A \) and \( B \) with respect to \( E \) is intuitively the most general way that \( E \) can be extended to an environment in which \( A \) and \( B \) can be recursively realized as identical expressions. It is possible that unifying \( A \) and \( B \) will make them unprintable. For example, the most general unifier of the expressions \( x \) and \( (0,x) \) with respect to the empty environment (\( \_ \)) is the environment (\( x \cdot (0,x) \)) in which \( x \) is bound to \( (0,x) \). This shows that in general it is possible for \( (\text{UNIFY } A \leftrightarrow B \leftrightarrow E) \) to be an environment in which the recursive realizations of \( A \) and \( B \) are identical but unprintable.

1.10 SUBSTITUTIONS

Some readers may be more familiar with the usual treatment of unification, which is developed in terms of the idea of substitutions. A substitution is a mapping from expressions to expressions which preserves proper names and the dotted pair structure. More precisely, a mapping \( S \) from expressions to expressions is a substitution if, and only if, it satisfies the two conditions:

\[
XS = X \quad \text{for all proper names } X, \\
(x,y)S = (XS),(YS) \quad \text{for all expressions } X \text{ and } Y.
\]

We denote the result of applying a substitution \( S \) to an expression \( X \) by the postfix notation: \( XS \), as illustrated above. An important property of a substitution is that its effect upon any expression is completely determined by its effect on the
variables (if any) which it actually changes. By listing those variables, each equated to its image under the substitution, we therefore give a complete description of the substitution. But the information in such a list of equations is just what is provided by an environment. The list of equations

\[ V_1 = A_1, \ldots, V_n = A_n \]

corresponds to the environment

\[ (V_1, A_1, \ldots, V_n, A_n) \]

and conversely. Indeed if \( S \) corresponds in this way to the environment \( E \), then the image \( XS \) of any expression \( X \) under \( S \) is just the expression \( (\text{REAL } X \ E) \). We write \( [E] \) for the substitution corresponding in this way to the environment \( E \). Thus we have

\[ X[E] = (\text{REAL } X \ E) \]

for all expressions \( X \) and environments \( E \). In this correspondence between environments and substitutions, the empty environment corresponds to the identity substitution (which transforms every expression into itself). Composition of two substitutions \( S \) and \( T \) yields the substitution \( ST \), which sends each expression \( X \) into the expression \( X(ST) = (XS)T \) obtained by first applying \( S \) to \( X \) and then \( T \) to the result. If \( S \) is \([E]\) and \( T \) is \([F]\), \( ST \) is \([L]\), where \( L \) is the list of all distinct bindings

\[ V.(V[L](F)) \]

where \( V \) is defined in \( E \) or in \( F \) (or both).

An environment \( E \) may be taken as a description not only of \([E]\) but also of the iterate of \([E]\). The iterate \( S^n \) of a substitution \( S \) is the "limit" of the series

\[ S, SS, SSS, \ldots \]

To find the image \( X(S^n) \) of an expression \( X \) under the iterate of \( S \), we repeatedly apply \( S \) to \( X \) until no further changes occur. That is, \( x(S^n) \) is the first expression in the series

\[ X, XS, XSS, XSSS, \ldots \]

which is the same as its predecessor. It turns out that if \( S \) is \([E]\) then \( X(S^n) \) is \((\text{RECREAL } X \ E) \). If \( S \) is \([E]\) then \( S^n \) is denoted by \([E]\). So we have
x[E] = (REAL X E)
x(E) = (REC REAL X E)

Now in terms of substitution mappings, a unifier of two expressions \( A \) and \( B \) is a substitution \( S \) which maps \( A \) and \( B \) onto the same expression:

\[
AS = BS
\]

and a most general unifier of \( A \) and \( B \) is a unifier \( U \) of \( A \) and \( B \) with the property that

\[
S = US
\]

for all unifiers \( S \) of \( A \) and \( B \).

Thus if \( U \) is an mgu of \( A \) and \( B \) and \( S \) is any unifier of \( A \) and \( B \) we have

\[
AS = AUS = BUS = BS \quad \text{and} \quad AU = BU
\]

so that the common expression onto which \( S \) maps \( A \) and \( B \) is obtainable by applying \( S \) to the common expression onto which \( U \) maps \( A \) and \( B \). The substitution \((UNIFY A \ B \ E)\) is the mgu of the two expressions \( A(E) \) and \( B(E) \). Thus UNIFY is given the two expressions to be unified in an indirect way.

1.11 IMPLICIT EXPRESSIONS

The way that the two expressions \( A(E) \) and \( B(E) \) are given to the UNIFY algorithm is indirect, in "unassembled" form. This idea of working with expressions not yet or possibly never fully assembled is used extensively in our system. It makes for computational economy and also for increased intelligibility. We think of the list \((A \ E)\) as an "implicit" way of giving the expression \( A(E) \). We say that \( A \) is the skeleton part, and \( E \) the environment part, of the implicit expression \((A \ E)\). For many purposes it is more convenient, as well as more economical, to deal with such "implicit expressions" than with the actual expressions themselves. This is particularly the case when \((A \ E)\) describes an unprintable expression even though both \( A \) and \( E \) are printable – as in the example previously mentioned when \( A \) is \( x \) and \( E \) is \((x,(0,x))\).
1.12 Instances

We often wish to consider, for some expression A, the various expressions AS, where S is a substitution. These are known as the instances of A. For example, the expressions

\[(\text{Divides } 17 \text{ by })\]
\[(\text{Divides } \text{(Plus } a \text{ by } b \text{) Times } 3 \text{ c})\]

are both instances of the expression \((\text{Divides } p \text{ by } q)\). The first of them is in fact a ground instance, since it contains no variables. In general we say that expressions which contain no variables are ground expressions; so a ground instance of A is an instance of A which happens to be a ground expression. Expressions which contain one or more variables are known as patterns. We often think of a pattern as a way of representing all of its instances.

1.13 Variants

In the role of a representative of all its instances a pattern is not unique. Other patterns - known as its variants - have exactly the same instances. For example, the expressions

\[(\text{Divides } p \text{ by } q)\]
\[(\text{Divides } x \text{ by } y)\]

have exactly the same instances. Each is a variant of the other. In general a variant of an expression A is an instance AS of A under a substitution which maps variables onto variables in one-to-one fashion. Such a substitution is called a variation, and is the only kind of substitution which has an inverse. If \(E\) is a variation then its inverse is \(E'\), where \(E'\) is obtained from \(E\) by interchanging the head and tail of each of its bindings. The compositions \(E[E']\) and \(E'[E]\) are then both the identity substitution.

In view of the identity of the set of instances of an expression with that of any variant of the expression, we often treat mutual variants as merely different ways of writing the same thing. However, in some of the computations involving patterns (such as the unification computation) it is sometimes necessary to take suitable variants of one's data beforehand.

To see why this is so, consider the problem of finding a pattern whose instances are exactly those which are instances of two given expressions, A and B.
For example, if $A$ and $B$ are the expressions

$$\text{Divides} \ (\text{Plus} \ x \ y) \ z \quad \quad \text{Divides} \ x \ (\text{Times} \ x \ y)$$

then among their common instances are the expressions

$$\text{Divides} \ (\text{Plus} \ 3 \ 4) \ (\text{Times} \ (\text{Plus} \ 3 \ 4) \ 6) \quad \quad \text{Divides} \ (\text{Plus} \ 0 \ 0) \ (\text{Times} \ (\text{Plus} \ 0 \ 0) \ (\text{Exp} \ x \ y))$$

and so on. We can get the first instance from $A$ by the substitution

$$x = 3, \ y = 4, \ z = (\text{Times} \ (\text{Plus} \ 3 \ 4) \ 6)$$

we can get it from $B$ by the substitution

$$x = (\text{Plus} \ 3 \ 4), \ y = 6$$

However, there is no single substitution $S$ such that $AS = BS = $ this common instance. The difficulty is the occurrence of the same variables in both $A$ and $B$. If we take a variant of $B$ which has no variables in common with those of $A$ — say, the expression

$$\text{Divides} \ p \ (\text{Times} \ p \ q)$$

which we shall call $C$ — then we can in fact find a pattern whose instances are exactly those common to $A$ and $B$. To do this we need only compute the expression

$$\text{RECREAL} \ A \ (\text{UNIFY} \ A \ C () )$$

or (which is the same)

$$\text{RECREAL} \ C \ (\text{UNIFY} \ A \ C () )$$

which is the "most general common instance" of $A$ and $C$ — and therefore also of $A$ and $B$.

Now the environment $(\text{UNIFY} \ A \ C ())$ is

$$\{ p, (\text{Plus} \ x \ y), z, (\text{Times} \ p \ q) \}$$

and so the required expression is

$$\text{Divides} \ (\text{Plus} \ x \ y) \ (\text{Times} \ (\text{Plus} \ x \ y) \ q)$$

Every expression which is an instance both of $A$ and of $B$ is an instance of this expression — and conversely. This example
illustrates the way in which the unification computation solves the general problem of constructing a pattern whose instances are precisely those which two given patterns have in common. Of course, when the two given patterns have no common instances, no such pattern exists. The UNIFY function detects all such cases by returning "IMPOSSIBLE" instead of an environment.
CHAPTER 2

LOGIC PROGRAMMING IN GENERAL

Logic programming is a technique for specifying computations by making assertions. No imperative constructs are used. The course of events during a logic computation is determined not by the programmer's control instructions (for there are none) but by the machine's pursuit of certain of the deductive consequences of the programmer's assertions. For example, the programmer might make the following assertions:

1. Drobný is a champion
2. Drobný is older than Rosewall
3. Rosewall is older than Goolagong
4. If $x$ is older than $y$ and $y$ is older than $z$, then $x$ is older than $z$
5. If $x$ was born before $y$ then $x$ is older than $y$
6. Kelly is a child of Goolagong
7. If $x$ is a child of $y$ then $y$ was born before $x$
8. Goolagong is female
9. Drobný is male
10. Rosewall is male
11. Rosewall is a champion
12. Goolagong is a champion
13. Connors is a champion
14. Borg is a champion
15. Connors is male
16. Borg is male
17. Borg was born before Connors
18. Connors was born before Kelly
19. Kelly is female
20. Everett is a champion
21. Everett is female
22. Everett was born before Connors

FIGURE 1

Some of these assertions are of particular facts; others are generalities involving the use of logical variables $x$, $y$, $z$. LOGIC is now capable of responding to queries about the "world" described by these assertions. In supplying answers to such queries it must in general deduce them from what it has been told (rather than merely look the answers up). For example, the
query:

which male champions are older than Kelly?

would elicit the answer

(Connors Borg Rosewall Ulobny).

That these persons are male and champions is explicitly given among the assertions, but that each of them is older than Kelly must be deduced. The deductions involved can, if desired, be examined by the user. For example, one could request:

* Explain the fourth answer

and LUGIC would respond with the following rationale:
To show: Droony is a male
Droony is a champion
Droony is older than Kelly

It is enough, by assertion 9.

to show: Droony is a champion
Droony is older than Kelly.

But then it is enough, by assertion 1.

to show: Droony is older than Kelly.

But then it is enough, by assertion 4.

to show: (there is a y:1 such that)
Droony is older than y:1
y:1 is older than Kelly.

But then it is enough, by assertion 2.

to show: Rosewall is older than Kelly.

But then it is enough, by assertion 4.

to show: (there is a y:2 such that)
Rosewall is older than y:2
y:2 is older than Kelly.

But then it is enough, by assertion 3.

to show: Goolagong is older than Kelly.

But then it is enough, by assertion 5.

to show: Goolagong was born before Kelly.

But then it is enough, by assertion 7.

to show: Kelly is a child of Goolagong.

But then it is enough, by assertion 6

to show: nothing.

End of explanation.

FIGURE 2
In the LOGIC system implemented within LOGLISP, the language of the queries, assertions and explanations is formalized and artificial. We shall shortly discuss the details of its design. Meanwhile, note that an explanation is essentially a proof, which proceeds in steps all of the same kind. At each step there is a "constraint list" of simple propositions, all to be shown true. Any variables in these propositions are considered to be existentially quantified by quantifiers placed at the beginning of the constraint list, and the constraint list itself is considered to be the conjunction of its members. The empty constraint list (i.e., the empty conjunction) is by convention true, so that if at some step the list has become empty, the proof is complete - there is nothing left to show. In general, each inference step consists of three stages:

1. The selection of a proposition A from the constraint list and of an assertion from the knowledge base whose conclusion B will unify with A.

2. The replacement of A in the constraint list by the constraints comprising the hypothesis (if any) of the selected assertion.

3. The application to the new constraint list of the most general unifier of A and B.

The notion of unification has been defined only for formal expressions, however, and so to make this account precise we must now recast it in terms of the formal language of LOGIC.

Let us now survey this formal language.

2.1 PREDICATIONS

The basic unit of the formal language is the predication. Predications are simple sentences of the subject-predicate form in which the predicate is written first and the subject second. The predicate P may be any "proper identifier" - that is, an identifier which is a proper name. (Recall that, in LISP, an identifier is an atom which is neither a numeral nor a string). The subject is a list of expressions called terms. Ground (i.e., particular) terms are essentially noun-phrases which denote things. A list A = (A1 ... An) of n ground terms denotes the n-tuple of things denoted respectively by the component terms A1, ..., An. Predicates denote properties of tuples. (Properties of tuples are often also called relations). The intuitive meaning of a ground predication with predicate P and subject A is the proposition that the tuple denoted by A has the property denoted by P. We write this formally as the list whose head is P
and whose tail is A.

Thus we might formally write:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Droony is a champion</td>
<td>(Champion Droony)</td>
</tr>
<tr>
<td>Droony is male</td>
<td>(Male Droony)</td>
</tr>
<tr>
<td>Droony is older than Kelly</td>
<td>(Older Droony Kelly)</td>
</tr>
<tr>
<td>Evert is female</td>
<td>(Female Evert)</td>
</tr>
<tr>
<td>Evert was born before Kelly</td>
<td>(Before Evert Kelly)</td>
</tr>
<tr>
<td>Kelly is a child of Goolagong</td>
<td>(Child Kelly Goolagong)</td>
</tr>
</tbody>
</table>

2.2 TERMS

A term may be either a variable, or a proper name, or a construction. Constructions have an operator-operand form. The operator (which may be any proper identifier) denotes an operation, and the operand may be any list of terms. When the construction is a ground expression, its operand denotes a tuple of things, in just the same way as does the subject of a ground predication. Constructions are indeed syntactically indistinguishable from predications. Their common syntactic form reflects an underlying unity in their semantics as applicative expressions. Each ground construction or ground predication can be understood as representing the result of applying some function to some argument. In the case of a predication this means construing a property or relation as a truth function, namely a function which yields as its result one or other of the two truth values, TRUE, FALSE. We write the construction with operator F and operand A = (A1 ... An) as the list

\[(F A1 ... An)\]

whose head is F and whose tail is A.

Ground predications, then, express facts and denote truth values. Ground terms express applicative descriptions and denote things. Both ground terms and ground predications have the same simple, systematic denotational semantics based on the applicative principle.

2.3 WORLDS

A world is a collection of facts — "everything that is the case" in that world. In logic programming a world is represented by a collection of ground predications. Given a collection \(W\) of ground predications as such a world, we can ask for what substitutions, if any, a given predication \(Q\) (whether ground or not) is "true in \(W\)". If \(Q\) is a ground predication, this is
simply the question whether $Q$ is a member of $W$. If $Q$ is in $W$, the answer is then: the identity substitution. If $Q$ is a predication pattern, however, this is not quite so simple a question, and we construe it to mean: for which substitution operations $E$ is the the instance of $Q$ under $E$ in $W$? For example, the world specified by the assertions in our earlier example is the set

\[
\begin{align*}
&\text{(Male Drobnj)} & \text{(Female Goolagong)} & \text{(Champion Drobnj)} \\
&\text{(Male Rosewall)} & \text{(Female Evert)} & \text{(Champion Rosewall)} \\
&\text{(Male Borg)} & \text{(Female Kelly)} & \text{(Champion Borg)} \\
&\text{(Male Connors)} & & \text{(Champion Connors)} \\
&\text{(Older Drobnj Rosewall)} & & \text{(Champion Goolagong)} \\
&\text{(Older Drobnj Goolagong)} & & \text{(Champion Evert)} \\
&\text{(Older Drobnj Kelly)} & & \\
&\text{(Older Rosewall Goolagong)} & & \\
&\text{(Older Rosewall Kelly)} & & \\
&\text{(Older Goolagong Kelly)} & & \\
&\text{(Older Borg Connors)} & & \\
&\text{(Older Borg Kelly)} & & \\
&\text{(Older Evert Connors)} & & \\
&\text{(Older Evert Kelly)} & & \\
&\text{(Older Connors Kelly)} & & \text{(Child Kelly Goolagong)} \\
\end{align*}
\]

**FIGURE 3**

With this world as $W$, if we ask what are the substitutions for which the predication

(Male $x$)

is true in $W$, we get four "solutions", namely:

\[
\begin{align*}
x &= \text{Drobnj} \\
x &= \text{Rosewall} \\
x &= \text{Borg} \\
x &= \text{Connors}
\end{align*}
\]

there being four ground instances of "(Male $x$)" in $W$, namely those corresponding to these four substitutions. More generally we can ask a question involving a conjunction of predications. If $Q_1, \ldots, Q_n$ are predications, we can ask of a world $W$

for what substitutions

is $(Q_1 \land \ldots \land Q_n)$ true in $W$?

- 2-b -
or more briefly:

what substitutions satisfy \((Q_1 \land \ldots \land Q_n)\) in \(W\)?

For example in the \(W\) of our example the question

what substitutions satisfy

\(\{(\text{Male } x) \land (\text{Champion } x) \land (\text{Older } x \text{ Rosewall})\}\)

in \(W\)?

has the answer

\(x = \text{Dröny}\)

since under this (but no other) substitution the conjunction becomes true in \(W\).

2.4 Queries

It is useful to introduce the formal notion of a query, based on the preceding discussion. A query is an expression of the form

\[(\forall x \ Q_1 \ldots \ Q_n)\]

in which \(Q_1 \ldots Q_n\) are predicates and \(x\) is an expression called the answer template of the query. The answer template may be any variable, any proper name, or any list of terms. The list \(Q = \{Q_1 \ldots Q_n\}\) is the constraint list of the query. For any world \(W\), such a query has an answer, which is a list of expressions. Each expression in this answer list is the instance of the answer template under a substitution which satisfies the constraint list \(Q\), that is, which transforms the conjunction \((Q_1 \land \ldots \land Q_n)\) into one which is true in \(W\). Thus the query

\[(\forall x \ (\text{Male } x) \land (\text{Champion } x) \land (\text{Older } x \text{ Rosewall})\)]

has the answer (in the world of our example)

\[(\text{Dröny})\]

since the substitution \(x = \text{Dröny}\) is the only one which satisfies the given constraint, while the query

\[(\forall z \ (\text{Female } z) \land (\text{Older } z \text{ Dröny}))\]

has the empty list
as its answer since there are no substitutions which satisfy the constraint

\[
\text{(Female z) (Older z Droony)}
\]

2.5 SPECIFYING A WORLD BY ASSERTIONS

It is not expected that one should have to specify a world by explicitly listing, as in FIGURE 3, all of its predications (although this would in principle be possible for a finite world). A world is specified indirectly, by giving a collection of assertions. An assertion has two parts: a conclusion, which is a predication, and a hypothesis, which is a list of predications. The hypothesis of an assertion can be the empty list, in which case the assertion is said to be an unconditional assertion, whereas an assertion whose hypothesis is nonempty is said to be a conditional assertion. An unconditional assertion whose conclusion is \( B \) is written

\[
B <-
\]

while a conditional assertion with conclusion \( B \) and hypothesis \( (A_1 \ldots A_n) \) is written

\[
B < - A_1 \ldots A_n
\]

A collection of assertions is called a knowledge base. Any such collection determines a world.

An unconditional ground assertion (i.e., a datum) \( B < - \) intuitively says that \( B \) is one of the facts in the world being described - "\( B \) is true". Recall that any assertion which is not a datum is a rule, which is a conditional ground assertion \( B < - A_1 \ldots A_n \) says that \( B \) is one of the facts in the world being described provided that \( A_1, \ldots, A_n \) all are - "If \( A_1 \) and \( \ldots \) and \( A_n \) are true then \( B \) is true". A rule which is an assertion pattern - an assertion containing one or more variables - has the same descriptive effect as would the set of all its ground instances. In general this means that an assertion pattern is in effect a universally quantified statement. If its variables are \( x_1, \ldots, x_k \) (say) then the assertion \( B < - A_1 \ldots A_n \) can be read

"for all \( x_1, \ldots, x_k \): if \( A_1 \) and \( \ldots \) and \( A_n \) then \( B \)"

Indeed, if some of the variables among the \( x_1 \) (say, \( z_1, \ldots, z_p \)) do not occur in the conclusion \( A \) while the rest (say, \( y_1, \ldots, y_t \)) do,
the assertion $\beta \leftarrow A_1 \ldots A_n$ may be more intuitively (but equivalently) read

"for all $y_1, \ldots, y_t$:

if there exist $z_1, \ldots, z_p$ such that $A_1$ and $\ldots$ and $A_n$ then $\beta$"

In the example of FIGURE 1 there are three such assertion patterns. All the other assertions in FIGURE 1 are data. FIGURE 4 shows the knowledge base of FIGURE 1 written in the formal notation.

1 (Champion Drooby) \leftarrow
2 (Older Drooby Rosewall) \leftarrow
3 (Older Rosewall Goolagong) \leftarrow
4 (Older x z) \leftarrow (Older x y)(Older y z)
5 (Older x y) \leftarrow (Before x y)
6 (Child Kelly Goolagong) \leftarrow
7 (Before y x) \leftarrow (Child x y)
8 (Female Goolagong) \leftarrow
9 (Male Drooby) \leftarrow
10 (Male Rosewall) \leftarrow
11 (Champion Rosewall) \leftarrow
12 (Champion Goolagong) \leftarrow
13 (Champion Connors) \leftarrow
14 (Champion Borg) \leftarrow
15 (Male Connors) \leftarrow
16 (Male Borg) \leftarrow
17 (Before Borg Connors) \leftarrow
18 (Before Connors Kelly) \leftarrow
19 (Female Kelly) \leftarrow
20 (Champion Evert) \leftarrow
21 (Female Evert) \leftarrow
22 (Before Evert Connors) \leftarrow

FIGURE 4

The knowledge base of FIGURE 4 completely determines the world of FIGURE 3, according to the following general definition.
The world determined by a knowledge base \( D \) is
the smallest set \( W \) of ground predicates which
satisfies the two conditions:

(1) If \( D \) contains the datum \( G \leftarrow \), then \( G \) is in \( W \).

(2) If \( G \) is a ground instance of a rule in \( D \)
and the predications in the hypothesis
of \( G \) are all in \( W \), then the conclusion
of \( G \) is in \( W \).

In effect, this definition describes a process which
infers \( W \) from \( D \) by a series of wholesale inference steps. First, by (1),
the process constructs outright the set \( W_0 \), which contains just
those ground predications which are conclusions of data in \( D \). Then by (2), in general, having constructed the set \( W_n \), this
process constructs \( W_{n+1} \) by adding to \( W_n \) the conclusion of every
ground instance \( G \) of every rule in \( D \), provided that every
predication in the hypothesis of \( G \) is in \( W_n \). Thus the process
constructs a series of bigger and bigger worlds

\[ W_0, W_1, \ldots, W_n, \ldots \]

which either ends (with a world that is the same as its
predecessor) or else continues indefinitely. The world \( W \) is then
the "limit" of this series, i.e., the union of all of the sets in
it; i.e., the smallest set which includes them all. Thus the
world \( W \) is determined by a knowledge base \( D \) through a "bottom up"
process of reasoning.

Given such a \( D \), we wish to be able to answer queries about its
world \( W \). In doing so we wish to avoid the brute force method of
generating \( W \) bottom up and searching it. It is much better,
given a query about \( W \) to reason "top down" about \( W \)'s contents
without actually constructing \( W \). This turns out to be possible
through the use of unification, built into a special inference
principle called LUSH resolution. This inference principle can
be applied very efficiently through the use of implicit
expressions, as we shall now see.
2.6 IMPLICIT CONSTRAINTS AND THEIR SOLUTIONS

By an implicit constraint we mean a pair \((Q, E)\) in which \(E\) is an environment and \(Q\) is a list of predicates. The expression \(Q(E)\) is the corresponding explicit constraint. Now let \(D\) be a knowledge base and let \(W\) be the world described by \(D\). We denote by \((\text{SOL } Q, E)\) the set of "solutions of \((Q, E)\) in \(W\)" — that is, the set of environments \(A\) which are extensions of \(E\) with the property that all of the predications in \(Q(A)\) are true in \(W\).

We wish to calculate \((\text{SOL } Q, E)\) from \((Q, E)\) and \(D\).

There are two cases to consider. The first case is when \(Q\) is empty. Then \((\text{SOL } Q, E)\) is simply the set whose only member is \(E\). Such a \((Q, E)\) is said to be solved.

The second case is when \((Q, E)\) is unsolved, i.e., when \(Q\) is nonempty. For this case we use LUSH resolution to represent the desired set as the union of one or more simpler sets.

2.7 LUSH RESOLUTION

For any unsolved constraint \((Q, E)\), any knowledge base \(D\), and any positive integer \(K\) not greater than the length of \(Q\), the set

\[
(\text{RES } Q, E, D, K)
\]

is a set (possibly empty) of implicit constraints called the \((D, K)\)-resolvents of \((Q, E)\). The interest of this set lies in the fact that we have:

\[
(\text{SOL } Q, E) = (\text{SOL } Q_1, E_1) \cup \ldots \cup (\text{SOL } Q_n, E_n)
\]

where \((Q_1, E_1), \ldots, (Q_n, E_n)\) are the \((D, K)\)-resolvents (if any) of \((Q, E)\). This equation holds for all the admissible values of \(K\) (however, the \((D, K)\)-resolvents will in general be different, for each value). In particular for some choices of \(K\) it may be that there are no \((D, K)\)-resolvents of \((Q, E)\). This then means that there are no solutions of \((Q, E)\) in \(D\), although other choices of \(K\) may delay the discovery of this fact by providing one or more \((D, K)\)-resolvents of \((Q, E)\).

2.8 THE CHOICE OF \(K\)

The computation of the set \((\text{RES } Q, E, D, K)\) involves a choice of the number \(K\). Accordingly we introduce a choice function \(SEL\). For each unsolved implicit query \((Q, E)\) and knowledge base \(D\) the number \((SEL Q, E, D)\) is a positive integer no larger than the
length of \( Q \). (In the LÚGLIC system as currently implemented in LÚGLISP, we take \((\text{SEL} \ Q \ E \ D) = 1\) throughout).

In general \( \text{SEL} \) might be expected to take into account the evidence available in \( Q \), \( E \) and \( D \) so as to make an informed choice with desirable pragmatic effects on the overall computation.

2.9 SPLITTING NONEMPTY LISTS

The purpose of the number \( K \) is to determine a decomposition of the list \( Q \) of the form: \( Q = L \# (A) \# R \), where \( A \) is the \( K \)th component of \( Q \).

In general we say that, for any list \( X \) and any positive integer \( K \) no larger than (LENGTH \( X \)), the \( K \)-decomposition of \( X \) is the triple \((L, A, X)\) such that \( A \) is the \( K \)th component of \( X \) and \( X = L \# (A) \# R \).

Thus when \( K = 1 \) we have \( L = () \), \( A = hX \), \( R = tX \).

2.10 SEPARATION OF VARIABLES

The computation of \((\text{RES} \ Q \ E \ D \ K)\) involves a further choice, namely of a variant \( D' \) of the knowledge base \( D \). \( D' \) must have the property that none of its assertions contains a variable which occurs in \((Q, E)\). This "standardizing apart" of the variables in the constraint from those in the assertions is necessary for the theoretical completeness of the resolution transformation. In the current implementation \( D' \) is selected automatically and represented implicitly and economically by techniques explained in Chapter 13.

2.11 DEFINITION OF \((\text{RES} \ Q \ E \ D \ K)\)

The set \((\text{RES} \ Q \ E \ D \ K)\) is the set of all implicit constraints of the form

\[(L \# H \# R \ (\text{UNIFY} \ A \ C \ E))\]

for which \( H \) is the hypothesis of an assertion in \( D' \) whose conclusion \( C \) unifies with \( A \) in \( E \), and where \((L, A, R)\) is the \( K \)-decomposition of \( Q \).
2.11.1 The Computation of \( (\text{RES} \cup \text{E} \cup \text{D} \cup \text{K}) \)

On the face of it, the entire knowledge base \( D \) must be searched in order to extract from it every assertion whose conclusion \( C \) will unify in \( E \) with the selected predicate \( A \) in \( D \).

Fortunately, this is not the case. For large \( D \) the cost would be prohibitive.

In fact it is possible to store \( D \) in such a way that only a relatively small subset of \( D \) need be searched. Note, first, that the predicate of \( C \) must be the same as that of \( A \) if \( C \) is to unify with \( A \) in \( E \). Accordingly, only those assertions need be considered whose conclusions satisfy this condition, and it is straightforward to partition \( D \) into subsets of assertions ("logical procedures") whose conclusions have the same predicate. Each logical procedure can be stored on the property list of the predicate and thus be retrievable in time essentially independent of the size of \( D \).

The data of each procedure can be further indexed on the basis of the various proper identifiers which occur in their conclusions. This is highly advantageous, since in order that a datum \( C \) unify with \( A \) in \( E \), \( C \) must in fact contain every proper identifier which occurs in \( A(E) \). This observation forms the basis of a quite selective retrieval technique. In practice it is found that large procedures consist mainly, if not entirely, of data, so that the retrieval technique frequently applies just when it will do the most good.

2.12 THE DEDUCTION CYCLE

The heart of the LOGIC system is the basic deduction cycle, which computes the set \((\text{SOL} \cup \text{E} \cup \text{D})\) for a given implicit constraint \((\text{U} \cup \text{E})\) and a given knowledge base \( D \).

The computation of \((\text{SOL} \cup \text{E} \cup \text{D})\) consists of the development of two sets of implicit constraints, SOLVED and WAITING. Initially, SOLVED is empty and WAITING contains the single constraint \((\text{Q E})\). These two sets are then subjected to an iterative transformation which corresponds intuitively to the construction of a "deduction tree" whose nodes are implicit constraints. The root of this tree is the implicit constraint \((\text{Q E})\). The successors (if any) of an unsolved node \((X Y)\) are the \((D K)\)-resolvents of \((X Y)\), for some admissible value of \( K \) which is selected by the function \((\text{SEL})\). The tips of the deduction tree are the solved nodes (if any) and the unsolved nodes (if any) which have no \((D K)\)-resolvents for the particular value of \( K \) assigned to them by the function \( \text{SEL} \). The output of the deduction cycle is the set of environment parts.
of the solved nodes of the tree.

As the tree develops, the solved nodes are collected into the set SOLVED, and the nodes which have not yet been processed are kept in the set WAITING. Thus the tree construction is finished when WAITING finally becomes empty.

The deduction cycle is the following three-step algorithm:

IN: let SOLVED be the empty set and
     let WAITING be the set containing only (Q E)

RUN: while WAITING is nonempty
     do  1 remove some constraint C from WAITING
          and let (X Y) be C
          2 if (X Y) is solved
              then add (X Y) to SOLVED
              else add the (D K)-resolvents of (X Y) to WAITING
                  where K = (SEL X Y D)

OUT: return the set of environment parts of SOLVED

In general (SOL Q E D) is computed by executing the deduction cycle and taking its output as the required set.

Several points are worth noting about the deduction cycle.

2.12.1 Failure Nodes: Immediate And Ultimate

An unsolved node of the deduction tree which has no solved nodes as descendants is known as a "failure". There are two kinds of failure. An immediate failure has no descendants at all — because it has no (D K)-resolvents for the particular value of K selected for it by SEL. An ultimate failure has one or more successors, but they too are failures — the entire subtree rooted in an ultimate failure consists of nothing but failures, and its tips are all immediate failures. It is an interesting problem to design implementations of the deduction cycle in which the subtrees rooted in ultimate failures are kept as small as possible without undue extra computation. Ideally, all failures would be immediate and would be recognised as such in constant (and short) time.
2.12.2 Nondeterminacy Of Deduction Cycle

There are several sources of nondeterminacy in the RUN step of the deduction cycle.

The most obvious of these are the explicit choices called for in steps 1 and 2. In both cases, the choice can be made uniformly and cheaply according to default criteria which are built into the system design. For example, in our own system the default criterion for the choice of $K$ is to choose always the value $K = 1$. In the PROLOG systems, the selection in step 1 is in effect ruled by a similar criterion - the first constraint $(X \ Y)$ is selected from a WAITING which is represented in effect as a list. (We have to say "in effect" because in fact the PROLOG systems handle WAITING dynamically in a backtrack mode of working which never explicitly realizes the whole list at once.)

The selection of the node $C$ in step 1 can (as in the PROLOG systems) be made according to the "depth first" criterion in which the younger members of WAITING are chosen before the older members. This may sometimes lead to the "depth first runaway" situation in which one or more nodes in WAITING are never selected because they are never the youngest. In practice other considerations (see the discussion below of the deduction window) preclude an infinite depth first runaway, but even the finite versions of it which are allowed by the deduction window may be thought undesirable. Avoidance of depth first runaway can be economically achieved by letting the selection in step 1 depend upon a quantity which can be computed once for all for each node when it is first generated. This quantity is the "solution cost" of the node.

2.12.3 Definition Of Solution Cost

The solution cost of a node $(X \ Y)$ is simply a heuristic estimate of the "cost" (in arbitrary units) of obtaining a solved descendent of $(X \ Y)$. Ordinarily we estimate this cost as the sum of \( LENGTH X \) and the depth of $(X \ Y)$, which is number of nodes preceding $(X \ Y)$ on its branch of the deduction tree. The user may select any linear combination of these two quantities as the cost estimate (see chapter 8). The selected node $C$ in step 1 is then always one whose solution cost is least. This method coincides with a breadth first development of the tree in the case when the solution cost of a node is taken to be its depth in the deduction tree.

We have also provided a "PROLOG" mode of operation in which the search is strictly depth first. (This mode does not incorporate any other special features of PROLOG.) Details concerning mode
selection are also given in chapter 8.

2.12.4 The Function SEL

The selection of the value of K in step 2 may well affect the total cost of computing the set of solved descendents of the selected node (X Y) – including the particular case when this set is empty and (X Y) is therefore a failure. However, the potential benefit of lowering this cost is offset by the expense of making the choice. The least costly selection criterion is that used by the PROLOG systems (and by our own system as its default criterion), namely, K = 1. We have not provided the normal user with any means of overriding the default value for K. The present discussion is intended to highlight an opportunity for the system designer to add a further layer of sophistication to the deduction cycle by making the choice of "which predication to resolve away" depend upon particular features of (X Y), rather than making it independent of all such features.

2.13 THE DEDUCTION WINDOW

Since in general the deduction tree can be infinite, it is necessary in some cases to truncate the deduction cycle and accept the resulting (perhaps incomplete) set of solutions as an approximation to the full set (which may be infinite).

It is desirable to manage this truncation gracefully and to provide the LOGIC user with some control over its details. This is the reason for the deduction window.

The deduction window is a collection of parameters which can be set in various ways by the user and which have default values which are used in the absence of user-provided alternatives.

The deduction window is discussed in more detail in Chapter 8.

Each parameter in the deduction window is used as an upper bound on an associated quantity measuring some feature of the deduction cycle. These quantities are TREESIZE, NODESIZE, ASSERTIONS, RULES and DATA.

At a given moment in the execution of the deduction cycle TREESIZE is the total number of nodes which have so far been generated. The RUN loop is terminated as soon as TREESIZE exceeds the bound set for it in the deduction window.

The implicit constraint (X Y) selected in step 1 of the body of the RUN loop is treated as an immediate failure (hence dropped
from \texttt{WAITING} without progeny) if \texttt{NODESIZE}(X Y), \texttt{ASSERTIONS}(X Y), \texttt{RULES}(X Y) and \texttt{DATA}(X Y) are not all within the bounds specified for them in the deduction window.

\texttt{NODESIZE}(X Y) is \texttt{LENGTH X}, the number of predicates in the constraint list \texttt{X} of (X Y).

\texttt{ASSERTIONS}(X Y) is the number of nodes which precede (X Y) on the branch of the deduction tree of which it is the current tip. This number is the same as the number of assertions invoked in its deduction. It is \texttt{0} for the initial node, and is \texttt{1} greater than that of its predecessor for each derived node.

\texttt{RULES}(X Y) is a quantity similar to \texttt{ASSERTIONS}(X Y), but reflects the classification of assertions into rules and data.

\texttt{RULES}(X Y) is the number of times a rule was invoked in the deduction of (X Y), and

\texttt{DATA}(X Y) is the number of times a datum was invoked in its deduction. We obviously have, for each (X Y) in \texttt{WAITING}, that:

\[
\texttt{DATA}(X Y) + \texttt{RULES}(X Y) = \texttt{ASSERTIONS}(X Y).
\]

Thus the deduction window serves as a truncation device which ensures that each particular execution of the deduction cycle will terminate. It provides the user with both a global (TREESIZE) and a local (NODESIZE, ASSERTIONS, RULES and DATA) cutoff control. All the bounds in the deduction window are set to system-defined default values in the absence of user-defined alternatives.

\subsection*{2.14 RECORDING DEDUCTIVE HISTORIES FOR LATER EXPLANATIONS}

The system can be asked to explain the logical genesis of some or all of the members of \texttt{SOLVED}. The deduction cycle so far described does not preserve the information which is needed to provide such explanations. At the option of the user, the deduction cycle can be modified to include provision for keeping a record of the "history" of each solved node. Such a history is essentially the branch of the deduction tree whose tip is that solved node. However, each node in the branch (after the first) must be labelled with the assertion which was invoked in deducing it from its predecessor node. A request to explain a given solved node can then easily be met by constructing from its history a text of the sort illustrated earlier in FIGURE 2.

The extra time and space needed to operate the deduction cycle in the historical mode are not so small as to be negligible. The
user therefore will probably decide to switch the deduction cycle into this mode only when the availability of explanations is worth the cost.

Explanations are discussed in more detail in Chapter 10.
CHAPTER 3

LOGIC PROGRAMMING IN LISP

LOGIC is related to LISP in two different ways.

First, it is implemented in LISP - that is, the LOGIC system consists of a collection of LISP functions which live in a LISP workspace and provide all the logic programming facilities described in this manual.

Second, LOGIC in a certain sense contains LISP. This means that the LOGIC programmer can invoke LISP from within LOGIC calls, by incorporating in assertions and queries pieces of text which can be handed over to LISP for processing. To understand how this works we need to discuss the notion of LISP-simplification.

3.1 LISP-SIMPLIFICATION OF LOGIC EXPRESSIONS

The expressions encountered by the LOGIC processor during the deduction cycle are terms and predications arising ultimately from the input constraint list and from the assertions used in constructing resolvents. However, some of these LOGIC expressions may also admit an interpretation as LISP programming constructs. In that case they may have a LISP value, or if not they may be capable of some simplification.

For example, the expression

\[(+ 3 (* 5 4))\]

is both a LOGIC term and a LISP construct. (We allow short names \(\ast\), \(-\), \(*\) and \(/\) for the LISP functions PLUS, DIFFERENCE, TIMES and QUOTIENT.) In the latter role, it is equivalent to, and can be replaced by, its "value", namely the numeral 23.

within any normal expression e to produce an expression which has the same meaning as e. Such replacements of expressions by others which are their values are basic equivalence-preserving transformations of ordinary computation as normally conceived. The presence of free variables does not invalidate this idea. Thus even though "a" has no LISP value the LISP construct

\[- 3-1 -\]
(+ a (* 5 4))
can be simplified; it is LISP-equivalent to and can be replaced by the simpler expression
(+ a 20)
even though the latter is not its "value" as in the first case. In general, an expression may well "reduce" to another expression even when it will not, in the usual sense, "evaluate" to a "value".

We refer to this process of replacing a LOGIC expression by one which is LISP-equivalent to it as "LISP-simplification". It can be done to any expression at any time and is always defined (but may be merely the identity transformation).

3.2 LISP DEFINITIONS

Certain reduction rules are built into LISP itself and come with the system whenever one sets up a LISP workspace. That is, certain identifiers are defined as denoting built-in LISP functions (CAR, CDR, PLUS, etc.) or as the keywords of built-in special forms (COND, SETQ, PROGN, etc.).

In addition to these built-in LISP definitions, a LISP workspace may contain further definitions made by the user. A collection of such user-coined LISP definitions indeed constitutes a LISP program.

3.3 REDUCTIONS, VALUES AND SIMPLIFICATIONS

The joint effect of the system- and user-imposed definitions in a LISP workspace is to determine a notion of "reduction".

Each LISP construct is either reducible or irreducible. If e is reducible, then there is another LISP construct called the reduction of e, to which e is LISP-equivalent and by which it may be replaced.

Accordingly we say that the simplification of an irreducible LISP construct e is e itself, while the simplification of a reducible construct e is the reduction of e. Thus simplification is always defined. It often coincides with evaluation — that is, the value of e and the simplification of e are often identical. But this is not always the case and the matter requires some care.
For example, the expression

\((\text{QUOTE (This is an S-expression)})\)

is evaluable and has as its value the expression

\((\text{This is an S-expression})\)

but it is irreducible and hence is its own simplification.

The expression

\((\ast (+ 3 4) (\% 5 x))\)

has no value (since its second argument expression contains an occurrence of a variable) but simplifies to the expression

\((\ast 7 (\% 5 x))\)

In general if a LISP construct e has an atomic value v which is a proper name, its simplification is also v. However, if v is not atomic, or is a variable, the simplification of e is the expression \((\text{QUOTE v})\), rather than the expression v. This is at first a somewhat surprising feature of the simplification notion. A little reflection soon shows its naturalness.

The intuitive notion of simplification is that it always yields an expression which cannot be further simplified - which is irreducible. Moreover, an expression e must be LISP-equivalent to the simplification of e - and this means that if e has the value v so must the simplification of e. These two considerations together require that the simplification of e be \((\text{QUOTE v})\) - the value of which is v - whenever the expression v might itself be evaluable and have a value w distinct from v. Only when v is a proper name is w necessarily identical with v.

Note that one effect of these definitions is to establish a convention for quoting atoms which differs somewhat from that used in LISP. As an example, the LOGIC expression

\((\text{MEMBER Borg (QUOTE (Connors Borg Evert)))})\)

is evaluable with value T, being analogous to the LISP expression

\((\text{MEMBER (QUOTE Borg) (QUOTE (Connors Borg Evert)))})\).

The utility of the LOGIC convention becomes apparent when one considers a predication such as
(Older Drooby Rosewall)

which, had LOGIC followed the LISP convention, would have to be written

(Older (QUOTE Drooby) (QUOTE Rosewall)),

a distinctly unpalatable form.

3.4 OBJECTS IN LOGLISP

Before proceeding into a detailed exposition of the interaction between LOGIC and LISP, we review the classification of LISP objects imposed by LOGIC. Recall that objects are either composite (constructions) or atomic. Atoms are identifiers, strings or numerals. Identifiers beginning with a lower case letter are variables, all others are proper identifiers. Proper identifiers, strings and numerals constitute the class of proper names. For technical reasons, we prohibit the use of the character ":" in variables, except for certain "subscripted variables" created by LOGIC, which will be explained later.

3.5 REDUCTION AND EVALUATION

Generally speaking, we consider that the "applicative" expression \[ e = (f \, e_1 \ldots \, e_n) \] is evaluable if \( f \) is the name of a function (defined in LISP) and \( e_1, \ldots, e_n \) are themselves evaluable. In this case we also say that \( e \) is reducible and that the reduction \( e \) is the value of \( e \), quoted when necessary as explained above. The latter is obtained by applying \( f \) to the values of the argument expressions.

The reduction of an arbitrary applicative expression, in general, is obtained by replacing occurrences of its outermost reducible subexpressions by occurrences of their reductions.

We proceed now to precise definitions of the notions of evaluability, reducibility, and reduction. We shall speak of the expressions in question as though they were explicitly represented. In fact, in the LOGLISP system we compute the reduction of an expression directly from its implicit representation, as economically as we can. The resulting reduction is also represented implicitly, with the same environment part.
3.5.1 Evaluable Expressions

Excepting certain special forms which are discussed below, we say that the expression \( e = (f \, e_1 \ldots \, e_N) \) is evaluable if

1. \( f \) is a proper identifier with property EXPR, SUBR, LSUBR or MACKS and \( e_1, \ldots, e_N \) are evaluable, in which case the value of \( e \) is obtained by applying \( f \) to the values of \( e_1, \ldots, e_N \).

2. \( f \) is a proper identifier with property FEXPR or FSUBR, in which case the value of \( e \) is the result of applying \( f \) to the expression list \( (e_1 \ldots \, e_N) \). (This is just the standard notion of "application" for FEXPR's and FSUBR's.)

A proper name is evaluable and its value is itself. Variables are not evaluable.

3.5.2 Reducible Expressions

Again excepting certain special forms, an applicative expression \( e = (f \, e_1 \ldots \, e_N) \) is reducible if

1. \( e \) is evaluable as above, in which case the reduction of \( e \) is the value of \( e \), \( v \), if \( v \) is a proper name, otherwise \( \text{QUOTE} \, v \)

or \( e \) is not evaluable, but

2. \( f \) is a proper identifier and one or more of the expressions \( e_1, \ldots, e_N \) is reducible, in which case the reduction of \( e \) is \( (f \, e_1' \ldots \, e_N') \), where \( e_i' \) denotes the simplification of \( e_i \).

Note that atoms, whether variables or not, are irreducible. Note further that expressions \( (f \, e_1 \ldots \, e_N) \) in which \( f \) is a variable, a number, or, indeed, anything except a proper identifier, are neither evaluable nor reducible. This convention may be justified intuitively on the ground that one doesn't know what to do in such a case. We could, in fact, extend the definitions to allow \( f \) to be a lambda expression, say, but have chosen not to do so for the present. Such an extension would complicate matters significantly with no great advantage in flexibility.
3.6 SPECIAL FORMS

In addition to the expressions just considered there are a number of special forms which are evaluable or reducible or both. Most of these are special forms of LISP.

Since the syntax of special forms is the same as that of applicative forms whose function designator is atomic, LISP users often slur over the distinction. It is, however, most important to remember that the LISP value of a special form is NOT obtained by "applying the function denoted by its head to the object denoted by its tail" — that being how the LISP value of an APPLICATIVE form is obtained.

There is a special process set up for obtaining the LISP value of each special form, to which a LISP interpreter switches on recognizing the keyword (COND, SETQ, PROGN, QUOTE, etc.) of that special form.

This little homily would not be necessary if the syntax of applicative forms were designed in the same way, and applicative forms were tagged as such by a keyword, say, APP. The high frequency of applicative forms in programs would make such a convention burdensome. No one wants to have to write

\[(APP + (APP * 3 4)(APP SIN 30))\]

Instead of

\[ (+ (* 3 4)(SIN 30))\]

3.6.1 Quotations

(QUOTE v) or (FUNCTION v)

These forms are always evaluable and never reducible. The value of either form is v. Both of these forms are "immune" to instantiation, that is, (QUOTE v)(E) is (QUOTE v), for any environment E, even though the entity v may contain occurrences of variables.

3.6.2 Listings

(LIST el \ldots en)

LIST is treated as though it were an ordinary function (an LSUBR, say) despite the fact that UCI LISP implements LIST by means of an FSUBR. This is just what one would naively expect.
3.6.3 Conjunctions

(AND e1 ... eN)

(AND) is evaluable and reducible with value (and reduction) T. (AND e) is reducible and its value and reduction are the value and reduction, respectively, of e. If e1 is evaluable and its value is NIL then (AND e1 ... eN) is evaluable and reducible and its value and reduction are NIL. If e1 is evaluable with a non-NIL value then the evaluable, reducibility, value, and reduction of (AND e1 ... eN) are those of (AND e2 ... eN). If e1 is not evaluable then (AND e1 ... eN) is reducible just in case e1 is reducible, and the reduction of it is (AND e1' e2 ... eN), e1' being the reduction of e1. All of this corresponds to LISP usage, the conjuncts being evaluated in order and only as far as necessary to determine the result.

3.6.4 Disjunctions

(OR e1 ... eN)

(OR) is evaluable and reducible with value (and reduction) NIL. (OR e) is reducible and its value and reduction are the value and reduction, respectively, of e. If e1 is evaluable and its value is non-NIL then (OR e1 ... eN) is evaluable and reducible and its value and reduction are the atom T. If e1 is evaluable with value NIL then the evaluable, reducibility, value, and reduction of (OR e1 ... eN) are those of (OR e2 ... eN). If e1 is not evaluable then (OR e1 ... eN) is reducible just in case e1 is reducible, and the reduction of it is (OR e1' e2 ... eN), e1' being the reduction of e1. All of this corresponds to LISP usage, the disjuncts being evaluated in order and only as far as necessary to determine the result.

3.6.5 Conditionals

(COND q1 ... qN)

(COND) is evaluable and reducible and its value and reduction are NIL. If q1 is (EO ... EM) then (COND q1 ... qN) is reducible (and possibly evaluable) just in case e0 is reducible or evaluable. If e0 is reducible but not evaluable then (COND q1 ... qN) is reducible with reduction (COND (EO' ... EM') ... qN) and is not evaluable. If e0 is evaluable with non-NIL value v, then (COND q1 ... qN) is reducible and its evaluable, reduction and value are those of (PRGN (QUOTE v) e1 ... eM). If e0 is evaluable with value NIL then the evaluable, reduction,
and value of (COND q1 ... qN) are those of (COND q2 ... qN). All of this conforms to customary LISP practice, since PROGN mimics the sequential evaluation of the expressions in a conditional "arm".

3.6.6 Sequential Compositions

(PROGN e1 ... eN)

(PROGN) is evaluable and reducible with value and reduction T. (PROGN e) is reducible and its evaluability, reduction, and value are those of e. If e1 is reducible but not evaluable then (PROGN e1 ... eN) is reducible with reduction (PROGN e1' e2 ... eN), e1' being the reduction of e1. If e1 is evaluable then (PROGN e1 ... eN) is reducible and its evaluability, reduction, and value are those of (PROGN e2 ... eN).

(PROGN e1 ... eN)

(PROGN1 e1 ... eN)

(PROGN1) is evaluable and reducible with value and reduction T. (PROGN1 e) is reducible and its evaluability, reduction, and value are those of e. If e1 is reducible but not evaluable then (PROGN1 e1 ... eN) is reducible with reduction (PROGN1 e1' e2 ... eN), e1' being the reduction of e1. If e1 is evaluable with value v then (PROGN1 e1 ... eN) is reducible and its evaluability, reduction, and value are those of (PROGN1 e2 ... eN (QUOTE v)).

(PROG loc s1 ... sN)

PROGS are neither evaluable nor reducible. There is no reasonable way to carry out a reduction of a PROG analogous to the reduction of PROG1 or PROGN expressions, and the necessity of assignment to the local identifiers of the PROG would lead to limited utility of such a construct, even if we were to define some notion of reducibility for PROGS. PROG may, of course, be used freely in the definitions of functions invoked from LOGIC.

3.6.7 Assignments

(SETQ ident e)

If e is evaluable with value v and ident is a proper identifier then (SETQ ident e) is evaluable with value v, and assigns v to ident. Otherwise (SETQ ident e) is reducible just in case e is reducible and its reduction is (SETQ ident e'), where e' is the reduction of e.
Note that assignment should be used with some caution in LOGIC, since the order in which assignments are performed is determined in part by the heuristic search methods, and thus is not readily predictable. Observe too that in order to obtain the LISP value of an identifier ident one must write \((\text{EVAL ident})\), not just 'ident'.

3.6.6 Selections

\[(\text{SELECTQ e (q1 . s1) \ldots (qN . sN) u})\]

Here \(s1, \ldots, sN\) are to be lists of expressions. The evaluation and reduction of the SELECTQ expression are basically the same as the evaluation and reduction of

\[(\text{COND ((EQ e q1) . s1) \ldots (EQ e qN) . sN) (T u))\]

except that reductions are expressed with SELECTQ and \(e\) is evaluated just once at the beginning. If one of the selection keys \(q1\) is a list \((l1 \ldots lM)\) then the corresponding COND predicate is

\[(\text{MEMQ e (LIST l1 \ldots lM)})\]

3.7 LOGLISP SPECIAL FORMS

The remaining special forms do not correspond to anything in conventional LISP. They provide means by which the LOGIC programmer may control the interaction between LOGIC and LISP in order to deal with various unusual circumstances.

\[(\text{LOGIC e})\]

Intuitively, the LOGIC form specifies that the result of evaluation is to be regarded as a logic expression rather than as an object, the effect most often being to suppress the normal quoting of non-atomic values.

More precisely, if \(e\) is evaluable with value \(v\) then \((\text{LOGIC e})\) is reducible; \((\text{LOGIC e})\) is evaluable according as \(v\) is evaluable, and the reduction and value of \((\text{LOGIC e})\) are the reduction and value of \(v\). If \(e\) is not evaluable, \((\text{LOGIC e})\) is reducible
according as e is reducible and the reduction of \((\text{LOGIC } e)\) is 
\((\text{LOGIC } e')\), where \(e'\) is the reduction of \(e\). Put differently,
when \(e\) is evaluable, we reduce or evaluate \((\text{LOGIC } e)\) by treating
the value of \(e\), \(v\), as a logic expression and reducing or 
evaluating \(v\). In practice it usually happens that \(v\) is neither 
evaluable nor reducible, in which case \((\text{LOGIC } e)\) reduces to \(v\).

\((\text{LISP } e)\)

The form \((\text{LISP } e)\) indicates that \(e\) is itself to be treated as the
value of \((\text{LISP } e)\). More precisely, \((\text{LISP } e)\) is never reducible;
it is always evaluable and its value is \(e\). \((\text{LISP } e)\) differs from
\((\text{QUOTE } e)\) in that \((\text{LISP } e)\) is subject to instantiation, thus
\((\text{LISP } e)(E)\) is \((\text{LISP } e(E))\).

\((\text{GROUND } e)\)

The form \((\text{GROUND } e)\) is similar to \((\text{LISP } e)\), but is evaluable only
if no variables occur in \(e\). More precisely, \((\text{GROUND } e)\) is never
reducible; it is evaluable if no variable occurs in \(e\), in which
case its value is \(e\).

\((\text{LOGIC-GR } e)\)

\((\text{LOGIC-GR } e)\) is precisely equivalent to \((\text{LOGIC } (\text{GROUND } e))\). It
follows that if any variable occurs in \(e\) then \((\text{LOGIC-GR } e)\) is
neither evaluable nor reducible. If no variable occurs in \(e\) then
\((\text{LOGIC-GR } e)\) is reducible and its evaluability, reduction, and
value are those of \(e\).

We shall illustrate a few applications for these forms. First,
consider

\((\text{LOGIC } (\text{SUBST } (\text{GROUND } x) (\text{GROUND } y) (\text{GROUND } z)))\)

which, as it stands, is neither evaluable nor reducible. Suppose
now we instantiate to obtain

\((\text{LOGIC } (\text{SUBST } (\text{GROUND } (+ (\text{VAR } A) 3))
(\text{GROUND } (\text{VAR } Q))
(\text{GROUND } (\leq (\text{VAR } Q) 10))))\)

where \text{VAR} is not the name of a LISP function. Since no variables
occur in the inner expressions these are evaluable, the
expression \((\text{SUBST ... })\) is evaluable, hence the whole reduces to

\((\leq (+ (\text{VAR } A) 3) 10)\)

The abbreviation \text{LOGIC-GR} is sometimes useful in connection with
FEXPR's. If $f$ is the name of a FEXPR or FSUBR then 
(LOGIC-$g_x$ $(f \text{ } e_1 \ldots \text{ } e_n)$) is evaIuable and reducible just in case
no variable occurs in any of the $e_i$'s, in which case the value and reduction of
(LOGIC-$g_x$ $(f \text{ } e_1 \ldots \text{ } e_n)$) are the value and reduction of
$(f \text{ } e_1 \ldots \text{ } e_n)$. This treatment of FEXPR's is sometimes a
useful alternative to the customary procedure described earlier.

3.8 SIMPLIFYING IMPLICIT CONSTRAINTS--THE FUNCTION SIMPLER

If $C = (Q \text{ } E)$ is an implicit constraint and $D$ is a knowledge base,
then $(\text{SIMPLER } C \text{ } D)$ is the implicit constraint which results from
simplifying one or more of the predications in $C$ and dropping
them if they simplify to "true". Specifically, $(\text{SIMPLER } C \text{ } D)$ is
the result of the following three-step algorithm:

1. let $(Q \text{ } E) = C$

2. while $Q$ is nonempty
   do let $(L \text{ } A \text{ } R)$ be the $(\text{SEL } Q \text{ } E \text{ } D)$-decomposition of $Q$
      and $B(E)$ be the simplification of $A(E)$
      
      if $B(E)$ is "true"
      then replace $Q$ by $L*R$
      else return $(L*(B)*R \text{ } E)$

3. return $(\text{NIL} \text{ } E)$

By "true" we mean any evaIuable expression whose value is not
NIL.

3.9 THE EXTENDED DEDUCTION CYCLE

In the actual LOGIC cycle of our LOGLISP system we include a step
of LISP-simplification in step 1 of the RUN loop. The full
description of the loop is then:

RUN: while WAITING is nonempty
   do 1. remove some $C$ from WAITING
       and let $(X \text{ } Y)$ be $(\text{SIMPLER } C \text{ } D)$

       2. if $(X \text{ } Y)$ is solved
          then add $(X \text{ } Y)$ to SOLVED
          else add the $(D \text{ } K)$-resolvents of $(X \text{ } Y)$ to WAITING
          where $K = (\text{SEL } X \text{ } Y \text{ } D)$

- 3-11 -
Note that the K selected in step 2 will be the same as that selected in the final iteration of SIMPLER. (Indeed, in LUGLISP this is obviously so since K = 1 uniformly; but it is true for every SEL function).

This means that the predication resolved away is the one which was just processed by SIMPLER and that it is therefore a LISP-irreducible expression. In particular it may be the expression NIL (i.e., the LISP representation of FALSE). In this case, there will be no resolvents forthcoming and (X Y) will therefore be a failure.

3.10 CONTROLLING REDUCTION

It is sometimes helpful to inform LUGIC that certain expressions are irreducible, either because it is known in advance that any attempt at reduction will be futile, or because reduction would for some reason be inappropriate. This can be accomplished by invoking the LISP function IRREDUCIBLE (which is an FSUBR) with a command of the form

```
(IRREDUCIBLE id1 ... idn)
```

id1, ..., idn being proper identifiers. Having done so, any expression of the form (idk ...) will thereafter be irreducible, regardless of the nature of its subexpressions. The effect of IRREDUCIBLE can be undone with

```
(REDUCTIBLE id1 ... idn)
```

(REDUCTIBLE is also an FSUBR). REDUCIBLE will not, however, repeal the system-mandated irreducibility of PROGS.

These matters are discussed further in Chapter 4, Creating Knowledge Bases.

3.11 SUBSCRIPTED VARIABLES

We have mentioned before that the variables occurring in assertions are, in effect, renamed before resolution so as to prevent unintended identification of variables in different assertions. This is accomplished by "subscripting" the variables in the assertions with appropriately chosen non-negative integers. Ordinarily this subscripting is hidden from the user, and is, in fact, performed implicitly and quite economically. Subscripted variables may, however, appear in answers to queries, and are routinely seen when monitoring deductions (see Chapter
9). In such cases, the subscripted variable is an identifier whose print name consists of an ordinary variable suffixed by one or more subscripts, each subscript consisting of a "::" followed by one or more digits. Examples are $x:7$ and $\text{date:3:17}$. Such variables, generated by the system, are the only variables which may contain "::".

3.12 UNIFICATION IN LOGLISP

There are a few points worth noting about the LOGLISP implementation of unification.

First of all, there is no check performed to see if a unification has created any cycles. Such a check would, if routinely made, be time-consuming. It appears that in normal LOGIC programming the check is unnecessary. Since unification is confined to the cases where the input expressions do not have variables in common, cycles can arise only if assertions or queries are formulated in certain abnormal ways.

The use of implicit representations throughout in any case makes it possible to work with infinite (cyclic) expressions as though they were finite (which in a suitable sense they are). It is only when a sophisticated user wishes to exclude such expressions from the domain of discourse that their detection becomes necessary.

Of course, any process (such as a naive recursive realization) which seeks to traverse every path in such an expression will run on indefinitely, and the user will want to avoid this situation. In designing LOGLISP we have assumed that any user deliberately creating such expressions will be sophisticated enough to use LISP to protect himself without being lectured at by us. We have further assumed that any user inadvertently creating such expressions will prefer to take the error messages or other indications of his mistake which LISP will provide - in place of the expensive LOGLISP overhead which would be needed to protect him from them.

3.12.1 Proper Names

Two proper names, say $\text{al}$ and $\text{a2}$, are considered to be unifiable iff (\textit{equal} al a2) or both are strings and have the same characters. Thus the condition for unifiability can be expressed as
(OR (EQUAL a1 a2)
   (AND (STRINGP a1) (STRINGP a2)
        (EQUAL (EXPLODE a1) (EXPLODE a2)))) .

This produces just the effect one wants, but note that distinct
identifiers with the same PNAME are not unifiable (it cannot be
the case that both are INTERNeD). The integer 1 unifies with the
floating-point number 1.0, on the other hand, and distinct
occurrences of the same floating-point value are unifiable.

3.12.2 Special Forms

Expressions in QUOTE and FUNCTION are treated specially.
(QUOTE el) unifies with (QUOTE e2) if and only if
(EQUAL (QUOTE el) (QUOTE e2)), and similarly for (FUNCTION f1)
with (FUNCTION f2). In addition, expressions of the form
(CONS el e2) may unify with expressions (QUOTE (a . d)). In
attempting to unify two such expressions any logic variables
appearing in (a . d) will be treated as "constants". Let us
define q[v] as follows: if v is a proper name then q[v] is v,
otherwise q[v] is (QUOTE v). In attempting to unify (CONS el e2)
with (QUOTE (a . d)) the unifier proceeds by attempting to unify
el with q[a], then, if successful, unifying e2 with q[d].
Variables in el and e2 will be bound to subexpressions of a and
d, QUOTEd when appropriate. Some examples will make things
clear. The expression

(CONS x y)

unifies with

(QUOTE (A B C))

with mgu x = A, y = (QUOTE (B C)). To take a more complicated
case,

(CONS (CONS F x) (CONS u v))

unifies with

(QUOTE ((F (A B)) C D))

with mgu

x = (QUOTE ((A B))), u = C, v = (QUOTE (D)) .

Expressions in QUOTE and FUNCTION are not otherwise unifiable.
It should be remarked that an expression like (F A QUOTE (B))
does not contain a quotation, merely an occurrence of the
constant QUOTE.
3.12.3 Variables As Tails

Ordinarily, an expression is either an atom or a list, but one may, in fact, introduce expressions which are composite but not lists. The only useful expressions of this class are those for which repeated CDR's eventually yield a variable, an example being \((\text{cdr } x) \cdot y\). We remark that the definitions of unification and resolution given in chapters 1 and 2 do not actually require that non-atomic expressions be lists. In a sense, there is really nothing special about a composite expression which is not a list, but such expressions are sufficiently unusual that a bit more discussion may be in order. Expressions of this sort are particularly useful in dealing with operators which take a variable number of arguments. To illustrate, the expression

\[ (+ x . y) \]

unifies with

\[ (+ u 7) \]

with mgu

\[ x = u, y = (7) \]

and also unifies with

\[ (+ (\text{cdr } u) 3) 7 (G A B)) \]

with mgu

\[ x = (\text{cdr } u), y = (7 (G A B)) \]

Thus a simple, but still rather flexible, rule for solving equations involving sums may be asserted by

\[ (\text{if} (- (\text{eq} (+ x . y) z) \text{assert} (- x (- z (+ . y)))) \]

3.12.4 The "Don't Care" Symbol

The identifier \#, called the "don't care" symbol, unifies with any expression whatever, but such a unification introduces no bindings. The effect is as though each occurrence of \# were replaced by a new variable not appearing elsewhere in the expressions to be unified, except that the implementation benefits from use of the don't care symbol.
To illustrate, the expression

\[(P \# x \#)\]

unifies with

\[(P \{F \#1\} \{G A\} \#)\]

with mu

\[x = \{G A\}\]  

3.13 REDUCTION OF EXPRESSIONS ENDING IN VARIABLES

The reduction of an expression \((f \text{ el } \ldots \text{ en } \cdot v)\) will now be explained. Such an expression is evaluable if and only if \(f\) is the name of a FEXPR or FSUBR, in which case the value is the result of applying \(f\) to \((\text{ el } \ldots \text{ en } \cdot v)\), an argument "list" with which few FEXPRS are prepared to cope. If \(f\) is a proper identifier, but not the name of a FEXPR or FSUBR, then the expression is not evaluable, but is reducible if any of \(\text{ el}, \ldots, \text{ en}\) are reducible, in which case the reduction is \((f \text{ el}' \ldots \text{ en}' \cdot v)\).

The sequentially evaluated LISP forms, those formed with AND, OR, COND, PROGN, PROG1 and SELECTQ, may also involve variable tails. Reduction proceeds as described before, stopping when a variable tail is encountered. Such expressions may be evaluable if the "evaluation path" avoids variable tails entirely.

3.14 SPECIAL RULES FOR RESOLUTION

The system "automatically" incorporates a number of special rules applicable to certain predicate symbols. In most cases these rules are just economical implementations of computations that could be achieved with ordinary assertions, but the rule for CONDITIONAL expressions constitutes a fundamental extension of the system, as it introduces a form of "negation as failure" (see the discussion in chapter 12, section 1.2). Application of any of the rules can be enabled or disabled at will by the user.

3.14.1 The Rules

Each of the rules is introduced by an informal, assertion-like description, followed by discussion and, in some instances, a nearly equivalent formulation with actual assertions.
3.14.1.1 Equations -

\( (= e_1 e_2) \leftarrow "e_1 \text{ and } e_2 \text{ are unified}" \)

The rule is the reflexive law of equality, and amounts to

\( (\leftarrow ( = x x)) . \)

This rule is not applicable to expressions of the form \( (\text{EQUAL } e_1 e_2) \), even though \( = \) and \( \text{EQUAL} \) denote the same function for purposes of simplification.

3.14.1.2 Conjunctions -

\( (\text{AND } p_1 \ldots p_N) \leftarrow p_1 \& \ldots \& p_N \)

Bearing in mind that \( (\text{AND}) \) simplifies to \( T \), the rule for \( \text{AND} \) amounts to

\( (\leftarrow (\text{AND } x \cdot y) \leftarrow x \& (\text{AND } y)) . \)

3.14.1.3 Disjunctions -

\( (\text{OR } p_1 \ldots p_N) \leftarrow p_i \text{ for } i = 1 \ldots N \)

Again, bear in mind that \( (\text{OR}) \) simplifies to \( \text{NIL} \). The rule for \( \text{OR} \) is practically equivalent to

\[ (\leftarrow (\text{OR } x \cdot y) \leftarrow x) \]
\[ (\leftarrow (\text{OR } x \cdot y) \leftarrow (\text{OR } y)) \]

except that resolvents for all of the disjuncts are obtained in one step.

3.14.1.4 Conditionals -

\( (\text{COND } (p_1 q_1) \ldots (p_N q_N)) \leftarrow p_k \& q_k \text{, for the first } k \text{ such that } p_k \text{ is provable} \)

Let us refer to the constraint from which \( (\text{COND } \ldots) \) was selected for resolution as the "original constraint". The control mechanism, in fact, begins by attempting to prove \( p_1 \). If it succeeds in doing so, it introduces a new resolvent consisting of \( q_k \) and the other predications of the original constraint in the environment which proved \( p_1 \). (Such a resolvent will eventually be produced for each proof of \( p_1 \), if the search continues so long.) If all attempts to prove \( p_1 \) terminate in failure then the control mechanism attempts to prove \( p_2 \), and so on. All of these
searches are carried out within the heuristic limitations imposed on the problem at the beginning. These searches are, moreover, carried out "in parallel" with searches for other solutions to the initial problem, in accordance with the standard heuristics, so that depth-first runaway will be avoided to the extent possible.

The "arms" of the Conditional expression need not have exactly two expressions. An arm of the form $(pk)$ is, for purposes of resolution, equivalent to $(pk \top)$, while an arm of the form $(pk \ qk \ \ldots \ qkm)$ is equivalent to $(pk \ (\text{PROGN} \ qk \ \ldots \ qkm))$.

This treatment of conditionals depends on a feature of the system not hitherto mentioned, namely the ability to associate a "continuation" with a node. The continuation is itself just a node of a somewhat special nature which is not itself available for computing resolvents. We write a node $C$ with continuation $K$ as "(C Continuation: $K$)". The resolvents of [(C Continuation: $K$)] are exactly the nodes [(R Continuation: $K$)] such that $R$ is a resolvent of $C$.

Let $(X \ Y)$ be a node whose resolvents are desired, let the selected component of $X$ be $P$, and suppose that $P(Y)$ has the form (COND $(pl \ q1) \ \ldots (pN \ qN)$). We obtain a "resolvent" which is

$$(((pl) \ Y) \ \text{Continuation:} \ ((\#\text{COND} \ q1) \ (p2 \ q2) \ \ldots) \ast X' \ Y))$$

where $X'$ consists of the unselected predications of $X$. Each proof of $pl$ generates a resolvent (NIL $Z$) with the same continuation, from which we "pop up" the continuation to obtain a resolvent $((q1) \ast X' \ Z)$. If and when all attempts to prove $pl$ fail, we pop up the continuation to obtain

$$((\text{COND} \ (p2 \ q2) \ \ldots (pN \ qN)) \ast X' \ Y)$$

which is added to WAITING.

Continuations are not usually printed when explaining answers or monitoring deductions, rather the fact that a node has a continuation is indicated by printing "[CONTINUED]". Users can instruct the system to print continuations in full by invoking the command (CONTINUATIONS ON). (CONTINUATIONS OFF) returns the system to the normal mode.
3.14.2 Controlling The Special Resolution Rules

All of the rules may be enabled or disabled by invoking functions of the form (AUTOx "flag") where flag may be either ON or OFF. The complete set of control functions for the resolution rules is

(AUTO= "flag")
(AUTOAND "flag")
(AUTO-OR "flag")
(AUTOCOND "flag")

Each function returns its argument. T or NIL may be used instead of ON or OFF, while these function behave like FEXPRs for atomic arguments, they evaluate non-atomic arguments, so one could, for example, type

*(AUTOAND (AUTO-OR OFF))

to disable both the AND rule and the OR rule. All of the rules are enabled by system initialization, hence by RESTORE (see the chapter on filling knowledge bases).
CHAPTER 4
CREATING KNOWLEDGE BASES

To create a knowledge base one begins with the empty knowledge base and adds assertions to it one at a time as explained below. Or one can extend an already existing knowledge base by installing it in a LOGLISP workspace and adding more assertions to it. The empty knowledge base is created by executing the command

(START)

which discards any assertions already present and initializes the LOGIC part of the workspace (without affecting the LISP definitions, if any, which the user may have set up).

4.1 ADDING AN ASSERTION TO THE KNOWLEDGE BASE

The assertion command

(\- \$ \- A1 ... An)

causes the assertion

\$ \- A1 ... An

to be added to the current knowledge base. The symbol \- is the assertion symbol. (It is pronounced "assert").

The arrow may be omitted. We shall often omit it in the examples in this manual.

4.1.1 Naming An Assertion

An assertion may be given a user-coined name. This is most conveniently done at the time the assertion is added to the knowledge base, using an extended assertion command. Execution of the extended assertion command

(\- N \$ A1 ... An)

adds the assertion \$ \- A1 ... An to the current knowledge base, as before, but also ascribes to it the name N. The user-coined name N may be any proper identifier. For example,
the following four transactions:

*(!-(Born Herbrand 12 February 1908))

ASSERTED

*(!-(Died Herbrand 27 July 1931))

ASSERTED

*(!- TURING1 (Born Turing 23 June 1912))

ASSERTED

*(!- TURING2 (Died Turing 7 June 1954))

ASSERTED

add four assertions to the knowledge base, the first two of which are anonymous, and the second two of which have been named respectively TURING1 and TURING2. Note that each assertion transaction is terminated by the message ASSERTED. If the assertion is ill-formed the message returned will be ERROR-Ignored, in which case the knowledge base is not altered by the transaction.

The assertions making up a knowledge base are organized into groups called procedures. All assertions in the knowledge base whose conclusions have the same predicate P are grouped together into a procedure which is called "the procedure P". It is thought of, intuitively, as the portion of the knowledge base which is relevant to establishing those facts in the world whose predicate is P.

Assuming that the knowledge base was empty before the above four assertions were added, the contents of the knowledge base now consists of two procedures, each containing two assertions.

By invoking the PRINTFACTS command (see the following Chapter on Displaying Knowledge Bases) the contents of the knowledge base can be displayed, its clauses organised into procedures. Thus:
PROCEDURE Born

(* PRINTFACTS *)

FACTS-ASSERTED

(PROCEDURE Born)

(\?- (Born Herbrand 12 February 1908))

(\?- TURING1 (Born Turing 23 June 1912))

(PROCEDURE Died)

(\?- (Died Herbrand 27 July 1931))

(\?- TURING2 (Died Turing 7 June 1954))

END

* 

If one adds an assertion with name N to a procedure which already has an assertion named N, then the name is removed from the older assertion and attached to the new one. A single proper identifier may, however, be used to name as many assertions as one likes, provided no two of these are in the same procedure.

4.2 THE FACTS MODE

A somewhat more convenient way of adding a succession of assertions is provided by the FACTS mode. By executing the command (FACTS) the user puts the system into the FACTS mode. This is simply a wait-read-assert cycle which expects successive assertions to be typed in. The prompt-message ASSERT: is printed by the system to signify its readiness to receive the next assertion. Thus the four assertions of our example could have been added by means of the following excursion through the FACTS mode:
*(FACTS)*

ASSERT: ((Born Herbrand 12 February 1908))

ASSERT: ((Died Herbrand 27 July 1931))

ASSERT: ((TURING1 (Born Turing 23 June 1912))

ASSERT: ((TURING2 (Died Turing 7 June 1954))

ASSERT:

DONE

Such a FACTS session is terminated by typing a semicolon in response to the ASSERT: prompt. It should be noted that the format in which an assertion B <- A1 ... An is typed for input to the FACTS mode is the list (B A1 ... An). The first item on this list may be the optional user-coined name, as illustrated above. The list format enables the system to accept inputs which are too large to fit all on one line. As in the standard LISP convention, the system reads line after line of typed input until a syntactically complete object has been formed. Thus in the following FACTS transaction the three-component assertion AGE-FORMULA is asserted on several lines, each of which after the first is prompted by a colon:

*(FACTS)*

ASSERT: (AGE-FORMULA

:  (Age person given-year a)

:  (Born person # # birth-year)

:  (= a ( - given-year birth-year)))

ASSERT:

DONE

The assertion AGE-FORMULA is now installed as the sole component of a procedure Age which computes a person's age in a given year by looking up the year in which that person was born and subtracting it from the given year. Note the use of the don't
care symbol (=) to match the day and month of birth, neither of which is needed for the deduction. The contents of the knowledge base may again be viewed by executing (PRINTFACTS):

#(PRINTFACTS)

FACTS-ASSERTED

(PROCEDURE Born)

(!- (Born Herbrand 12 February 1908))

(!- TURING1 (born Turing 23 June 1912))

(PROCEDURE Died)

(!- (Died Herbrand 27 July 1931))

(!- TURING2 (Died Turing 7 June 1954))

(PROCEDURE Age)

(!- AGE-FORMULA (Age person given-year a)
   <- (Born person # year birth-year)
   & (= a (- given-year birth-year)))

END

The "<-" and "&" appearing in AGE-FORMULA are simply "syntactic sugar" intended to assist the reader in perusing complex assertions. These may also be typed in assertions given to !- or FACTS, but we usually don't bother to do so.

An ill-formed assertion typed to FACTS will be ignored, and a message will be typed to inform the user.

4.3 ADDING ASSERTIONS FROM LISP FUNCTIONS

The assertion function !- is just a LISP FEXPR, and as such may be invoked by any LISP function. LISP programmers will usually find it more convenient, however, to use the "SUBK-type" function ASSERTCLS of one argument, whose value should be a list as might be typed to FACTS (or appear as the tail of an invocation of !-). If the assertion is well-formed it will be added to the knowledge base and ASSERTCLS will return NIL. If the assertion is ill-formed it is ignored and ASSERTCLS returns ERROR.
4.4 ORDER OF ASSERTIONS IN THE KNOWLEDGE BASE

The order of the assertions within a single procedure is first the data, if any, in the order in which they were entered, then the rules of the procedure, in the order in which they were entered. This is the order in which the assertions are printed by PRINTFACTS.

The order of the procedures in the knowledge base is the order in which assertions for the procedures were first entered. This also is the order used by PRINTFACTS. It should be noted that the order of procedures is frequently changed by editing (see Chapter 6).

4.5 DECLARING ATTRIBUTES OF PROPER IDENTIFIERS

One may ascribe various attributes to proper identifiers in order to influence the operation of LOGIC. An example is IRRED, the attribute which indicates irreducibility, and others will be introduced later. Several LISP functions are provided for declaring such attributes.

(PROCEDURE "id" "atl" ... "atn") [FEXPR]
(CONSTANT "id" "atl" ... "atn") [FEXPR]

Either of these sets the attributes of the proper identifier id to (atl...atn), having first erased any previous attributes. Thus (PROCEDURE id) declares that id has no special properties. PROCEDURE is intended for use with predicates, CONSTANT for use with other identifiers, but both names in fact invoke the same function. PRINTFACTS will display attributes of predicates and constants as invocations of these functions.

As mentioned earlier, alternative means are provided for declaring identifiers irreducible.

(IRREducible "idl" ... "idn") [FEXPR]

declares idl,...,idn to be irreducible (attribute IRRED), retaining any previous attributes.

(REducible "idl" ... "idn") [FEXPR]

erases the attribute IRRED from idl,...,idn, without affecting other attributes.

- 4-6 -
The argument L should be a list of proper identifiers. Each function has the same effect as the corresponding EXPR, for the identifiers listed.

One may also declare attributes of identifiers while in FACTS mode. To do so, one types a line of the form

```
ASSERT: id at1 ... atn
```

in response to the prompt "ASSERT:". The effect is to declare all at1,...,atn as attributes of id in addition to any previous attributes. FACTS uses LINEREAD, so one can type such declarations over many lines if it should ever seem necessary.

The attributes used by LOGIC are IRRED, ONERES and NOHIST. Other attributes may be declared and will be recorded, but have no effect on the operation of the system.

### 4.6 Subscripted Variables in Assertions

Although it rarely happens in practice, one might attempt to enter an assertion containing subscripted variables. For technical reasons, subscripted variables may not appear in the knowledge base. If one does attempt to enter an assertion containing subscripted variables, or variables in the sequence genvar001, genvar002, ..., the system will rename such variables, using variables genvar001, so that the assertion which results in the knowledge base is a variant of the assertion which was entered, and has no subscripted variables.
CHAPTER 5
DISPLAYING KNOWLEDGE BASES

Various commands are provided for viewing the contents of a knowledge base.

5.1 DISPLAYING THE ENTIRE CONTENTS OF A KNOWLEDGE BASE

The command (PRINTFACTS) causes the system to print out a display of the entire current knowledge base. The display is organised into groups of assertions preceded by the message FACTS-ASSERTED. Each group of assertions constitutes a (logical) procedure. That is to say, the header of every assertion in the group has the same predicate (say, P). The predicate P is used as the name of the procedure, and the group of assertions is accordingly preceded by the line: (PROCEDURE P), or, if P has special attributes AT1,...,ATn, the line: (PROCEDURE P AT1 ... ATn).

The constituent assertions of the procedure P are then displayed in the form of assertion commands. The order in which the assertions appear in the display is data first, then rules, in the order in which they were asserted within each class. The display is terminated by the message END.

5.2 DISPLAYING A PROCEDURE

The command (PRINTFACTSOF P) displays the procedure P in the same style as that of the (PRINTFACTS) display. If one wishes to print several procedures one types (PRINTFACTSOF P1 ... PN).

Further, the standard function PP (synonymous with GRINDEF) has been altered to print logic procedures in addition to the properties which it ordinarily prints. These too are in PRINTFACTS format.

The command (PRLENGTH P) returns the number of assertions in the procedure P:

*(PRLENGTH Born)*

2.

*
5.3 DISPLAYING THE SET OF DEFINED PREDICATES

The command (PREDICATES) returns a list of the predicates for which logic procedures are defined in the current knowledge base. With the example of the preceding chapter we have:

*(PREDICATES)

(Born Died Age)

The command (CONSTANTS) returns a list of the constants which have been declared. These are proper identifiers other than predicates which have special LOGIC attributes.

5.4 DISPLAYING DATA IN WHICH A GIVEN PROPER IDENTIFIER OCCURS

It is often convenient to be able to retrieve and display the set of data in a given knowledge base in which a given notion occurs explicitly. Such a set in some sense corresponds to what the knowledge base says about that notion in a direct way. The command (PRINTCREFSOF C) displays all data in which the constant C appears somewhere. These assertions are organised into groups by their procedure name, but the entire procedure is not necessarily shown (only those of its assertions whose headers actually contain C).

5.5 RETRIEVING A PROCEDURE AS A LIST

The procedure P may be obtained as a LISP data object, namely, as the list of its constituent assertions. This list is returned as the value of the command

(ASSERTIONSOF P)

Each assertion B <- A1 ... An in the procedure is represented as the list (B A1 ... An). If the assertion has the user-coined name N then it is represented as the list (N B A1 ... An). For example, (ASSERTIONSOF Born) returns the list

(((born Herbrand 12. February 1908.))
(TURING1 (born Turing 23. June 1912.)))

The result of ASSERTIONSOF shares no list structure with the internal representation of the knowledge base, thus list-altering operations such as RPLACA and RPLACU performed on this list will have no effect on the knowledge base.
5.6 RETRIEVING INDIVIDUAL ASSERTIONS

One may display one or more individual assertions using a command of the form

(PRINTNA dsg1 ... dsgn) [EXPR]

where dsg1,...,dsgn are "assertion designators". In its simplest form an assertion designator is just an assertion name, but more elaborate forms may be used to resolve possible ambiguities, and indeed to designate any assertion in the knowledge base, whether named or not.

The possible forms for assertion designators are shown below. Here "pred" denotes a predicate, 'name' an assertion name, and 'numb' a positive integer.

name
(pred name) (possibly ambiguous)
(pred numb) (possibly ambiguous)
(pred Datum name)
(pred Rule name)
(pred Datum numb)
(pred Rule numb)

As indicated, some of these forms may be ambiguous, depending on the state of the knowledge base. Where a number is given, it specifies the ordinal position of the assertion within its class (rules or data) in the indicated procedure. The concise form (pred numb) is ambiguous if the procedure for 'pred' has both a datum 'numb' and a rule 'numb'. The forms (pred Datum name) and (pred Rules name) are redundant, and either is treated as though it were (pred name).

PRINTNA prints the indicated assertions and returns the list (dsg1...dsgn). An appropriate error message will be printed for any designator which is either ambiguous or fails to designate an assertion.

One may also retrieve an individual assertion as a list. The function

(ASSERTION dsg) [EXPR]

returns a list representing the assertion designated by (the value of) its argument, if there is one, NIL if the argument fails to designate an assertion. Assertions are represented in the same manner as with ASSERTIONSOF.
CHAPTER 6
EDITING KNOWLEDGE BASES

The resident editor of the LISP system has been extended so as to allow the editing of knowledge bases in essentially the same style as is used to edit LISP functions and data objects. The edit command EDITA is used to enter the editor when LOGIC editing is to be done. The following editing session will illustrate the way this works. We will use the editor to attach names to the (at present) anonymous assertions in the current knowledge base, and to change the name AGE-FORMULA to AGE-RULE.

*(EDITA Born)*

EDIT
#P

((E ) (TURING1 & ))
#PP

(((Born Herbrand 12 February 1908))
 (TURING1 (Born Turing 23 June 1912)))
#1 PP

((Born Herbrand 12 February 1908))
 #(-1 HERBRAND1) PP
 (HERBRAND1 (BORN Herbrand 12 February 1908))
#OK

Born
*(EDITA Died 1 (-1 HERBRAND2) PP)*

(HERBRAND2 (Died 27 July 1931))

Died
*(EDITA Age 1 (1 AGE-RULE) PP)*

(AGE-RULE (Age person given-year a)
 (born person # # birth-year)
 (= a (- given-year birth-year)))

Age
*
This editing has produced the desired changes, as may be seen if we display the resulting knowledge base:

*(PRIFNFACTS)
FACTS-ASSERTED

(PROCEDURE Born)

(\#- HERBRAND1 (Born Herbrand 12 February 1908))
(\#- TURING1 (Born Turing 23 June 1912))

(PROCEDURE Died)

(\#- HERBRAND2 (Died Herbrand 27 July 1931))
(\#- TURING2 (Died Turing 7 June 1954))

(PROCEDURE Age)

(AGE-RULE (Age person given-year a)
<-(Born person # # birth-year)
& (= a (- given-year birth-year)))

END
* 

which is what we wanted. If one wishes to edit several procedures simultaneously one types *(EDITA (P1 ... PN)). In this case one edits the assertions for all of the procedures as a single list. For example:

*(EDITA (Born Died))

EDIT

#{

(\#HERBRAND1 &) (TURING1 &) (HERBRAND2 &) (TURING2 &)
#OK

(Born Died)

* 

- 6-2 -
As with the LISP edit functions EDITF and EDITV, one may also specify one or more editor commands after the predicate (or list of predicates) to be edited. In such cases the commands are performed and the editor returns without further interaction with the user. For this reason it is very important that one remember the parentheses when specifying several procedures.

6.1 REMOVING PROCEDURES FROM THE KNOWLEDGE BASE

If one wishes to remove one or more procedures Pl, ..., PN from the current knowledge base one invokes the command (ERASEP Pl ... PN).

6.2 IMPOSSIBILITY OF UNUSUAL EXITS FROM EDITA

During an editing session under control of EDITA, the procedures being edited are temporarily removed from the knowledge base. In order to prevent accidental loss of these procedures the system has been arranged so that unusual exits from EDITA are impossible.

A "usual" exit occurs when the user types OK or when all of the commands specified in an invocation of EDITA are completed without error. An attempt at an unusual exit may result from an error, from the editor command STOP, or from the user typing ACAD. Whenever an unusual exit is attempted the system types the message

Editing of assertions interrupted. Exit with OK,

and leaves the user in EDITA, positioned at the beginning of the list of assertions being edited. This list will reflect any alterations made before the attempted exit.

6.3 EDITING INDIVIDUAL ASSERTIONS

When appropriate one may edit an individual assertion, using the functions we are about to describe. This is often more convenient than accomplishing the same result with EDITA, and incurs much less overhead when the assertion appears in a large procedure.

(EDITNA "name", "coms") [FEXPR]

edits the assertion "name" with optional edit commands "coms". The name may in fact be any assertion designator, in the manner of PRINTNA, and inappropriate designators lead to an error message and immediate return.
When using either of these functions one actually edits a copy of the original assertion. The knowledge base is not changed until one exits from the editor with OK, and then only if the resulting assertion is well-formed. If the new assertion is ill-formed a message to that effect is printed and one is returned to the editor, looking at the altered copy.

If the modified assertion belongs to the same procedure as the original, and has not been changed from a rule to a datum, or vice-versa, then the modified assertion will occupy the same position in the procedure as the original. In any other case the original assertion is deleted from the knowledge base, then the modified assertion is added (as a new assertion).

Abnormal exits from the editor are possible when editing an individual assertion. If such an exit does occur one is returned directly to the top level and the knowledge base is not altered.

6.4 DELETING ASSERTIONS

A number of special functions are provided for deleting selected assertions from the knowledge base. These are often more convenient to use than EDITA, and are considerably more efficient. In most cases we provide both FEXPRs for use from the terminal, and EXPRs intended to be called from LISP functions.

(DELETE "dsgl" ... "dsgr"
)  [FEXPR]

deletes the assertions designated by dsgl,...,dsgr. Inappropriate designators are ignored, and DELETE returns a list of designators for assertions which were actually deleted.

(DELETE dsg)  [EXPR]

deletes the assertion designated by dsg, if there is one. DELETE returns T if an assertion was deleted, NIL otherwise.

(DELETE "assrn")  [FEXPR]
(DELETE assrn)  [EXPR]

Each of these functions deletes the assertion which is EQUAL to the specified assertion, if there is one. Assertion names and
"sugar" in assrn are ignored in determining equality. Either function returns T if the specified assertion was found and deleted, NIL otherwise.

The following examples illustrate the use of DELETE= and DELETEA= in the context of the example used earlier:

*(DELETE= (born Turing 2 June 1912))
T

*(DELETEA= '((Died Turing 23 October 1954)))
T

The effect of these is to delete the two assertions giving dates of birth and death for Turing. Note that when using these to delete rules the variables specified in the parameter to DELETE= or DELETEA= must be the same as those appearing in the knowledge base.

(DELETEA= "assrn")
(DELETEA+ assrn)

The argument specifies an assertion, as with DELETE= and DELETEA=. All assertions which are instances of the specified assertion are deleted. Either function returns T if at least one assertion was deleted, NIL otherwise. The predicate of the header of the argument must be a proper identifier, not a variable.

(DELETE= "assrn")
(DELETE+ assrn)

These functions are like DELETEA and DELETEA+, except that only rules will be deleted.

(DELETED= "assrn")
(DELETED+ assrn)

The same, except that data are deleted.
CHAPTER 7
FILING KNOWLEDGE BASES

The current knowledge base may be preserved in a file by the
LOGIC primitive SAVE. The command (SAVE N) creates on the disk a
file named N. N is a user-coined name of no more than six
characters of which the first is an upper-case letter. When
this work is completed the message DONE is printed.

The file created by SAVE is written on the user's file structure
DSK: as conventional text, though not so prettily formatted as
with PRINTFACTS. An extension may be specified with the file
name, in which case the form is (SAVE (NAME , EXT)), following
the usual LISP convention. An extension is supplied only when
explicitly specified.

7.1 RESTORE AND LOADLOGIC

A file which has been created by a command (SAVE N) may be later
read into primary storage by means of a (RESTORE N) or a
(LOADLOGIC N) command. The command (RESTORE N) restores the
knowledge base to its contents as of the time the (SAVE N)
command which created the file was executed. The command
(LOADLOGIC N) adds the procedure clauses in the saved knowledge
base N to the procedure clauses in the current knowledge base.
RESTORE first clears out the current knowledge base whereas
LOADLOGIC does not.

File names with extensions are specified just as for SAVE. The
file in question must be found on file structure DSK; but the
project-programmer number of the area from which the file is to
be read may be specified separately. The most convenient form is
(RESTORE [proj,prog] file). Note that LOGLISP normally expects
numeric input in decimal, while ppn's are usually written in
octal. One way around this small difficulty is to use a command
such as:

*(RESTORE (7330,210) PLACES)
DONE
*
7.2 ADDTO AND BUILD

The existing primitives of LISP's filing system have been adapted appropriately for use in LOGLISP. In LISP the command \( \text{ADDTO } N \text{ Pl \ldots Pk} \) adds the LISP objects named Pi, \ldots, Pk to the file named \( N \) (and opens a new file named \( N \) if one does not already exist). In LOGLISP the effect of ADDTO is extended so that the objects Pi can also be LOGIC objects, namely logical procedures. Thus the command

\[
\text{(ADDTO BIOG Born Died)}
\]

creates (assuming it does not already exist) a file named BIOG and records that its members are the logical procedures Born and Died. The command

\[
\text{(ADDTO BIOG Age)}
\]

then extends the description of the file BIOG by recording that the logical procedure Age is also a member. Once a file has been opened and described by one or more ADDTO commands, it may be constructed and written on the disk by means of the BUILD command. The command \( \text{BUILD N} \) writes out onto disk storage the current members of the file \( N \) in their current condition.

7.3 DSKIN

Files created and stored using the ADDTO and BUILD primitives may be read into primary storage from the disk by means of the DSKIN primitive. Thus if the file BIOG had been previously written on the disk by execution of the command \( \text{BUILD BIOG} \), it would be read into primary storage by execution of the command \( \text{DSKIN BIOG} \). DSKIN is analogous to LOADLOGIC in that no prior clearing of the knowledge base currently in primary storage takes place before the asserting of the procedure clauses in the file. Thus by "disking in" several files of logical procedures one may build up a knowledge base containing them all. The format of files created by BUILD is a series of executable commands. Thus under the current assumptions as to the description of BIOG and the contents of its members, the command \( \text{BUILD BIOG} \) would create the file:
(SETQ IBASE 10.)

(SETQ DSKINATOM (QUOTE BIOG))

(DEFPROP BIOG (BIOG . LSP) FILENAME)

(DEFPROP BIOG (Born Died Age) MEMBERS)

(PROCEDURE Born)

(\-- HERBRAN01 (Born Herbrand 12. February 1908.))

(\-- TURING1 (Born Turing 23. June 1912.))

(PROCEDURE Died)

(\-- HERBRAN02 (Died Herbrand 27. July 1931.))

(\-- TURING2 (Died Turing 7. June 1954.))

(PROCEDURE Age)

(\-- AGE-RULE (Age person given-year a)
< (Born person # # birth-year)
 & (= a (= given-year birth-year)))

The command (PROCEDURE P) declares P to be a logical procedure and may be used to record further pragmatic information about P as explained in the Chapter on Interacting with LOGLISP. The DEFPROP commands are LISP acts of definition, recording on the property list of "BIOG" the information describing BIOG's properties as a filename. The two SETQ commands create the appropriate LISP environment for the execution of the rest of the commands in the file.

The decimal points appearing in the numerals in this example indicate that the values in question are to be interpreted in decimal, regardless of the value of IBASE. The resulting numbers are integers, not floating point.
CHAPTER 6
DEBUCING ANSWERS TO QUERIES

The deduction machinery of LOGIC is invoked by the deduction commands: ALL, ANY, THE, and SETOF. The first three are LISP FSUBR's which may conveniently be invoked from the terminal or within assertions. SETOF is a SUBR intended for use by LISP programs.

8.1 ALL

The command (ALL X Cl ... Cn) returns a list of simplifications of the instances of the answer template X with respect to all of the environments which satisfy the constraint (Cl ... Cn) in the current knowledge base. (These environments are called the solutions of the constraint (Cl ... Cn).)

The answer template X may be a variable, an atom not a variable, or a list of expressions. We emphasize that the answers returned are the expressions (or lists of expressions) obtained by simplifying the instances of the answer template in the solution environments, not the values of those expressions, which need not, after all, be evaluable.

8.2 ANY

The command (ANY K X Cl ... Cn) behaves in a similar manner, except that no more than K instances of X are returned from among those which the corresponding ALL command would return. K is expected to be a nonnegative integer.

8.3 THE

The command (THE X Cl ... Cn) returns the sole member of the list (ANY l X Cl ... Cn), if there is one, and is intended for use only in contexts where it is known that exactly one solution exists. If no solution exists for the given constraint, THE returns the identifier No-solutions-found.
8.4 SPECIFYING THE DEDUCTION WINDOW

The constraints appearing in invocations of ALL, ANY and THE need not consist entirely of predications. They may also contain limit specifications which determine the deduction window to be used. The form of a limit specification is

\[
\text{Limit: Value}
\]

where "Limit:" is one of TREESIZE:, NODESIZE:, ASSERTIONS:, RULES:, DATA:, and "Value" is a number, the identifier INF (denoting infinity) or a non-atomic expression whose LISP value is a number or INF. These values determine bounds for the corresponding parameters of the search window. Thus one might, in the context of the "tennis" example of chapter 2, ask for

\[
(\text{ALL } x \text{ (Champion } x \text{) (Male } x \text{) (Older } x \text{ Pete) RULES: 5})
\]

to obtain the set of all those who can be deduced to be male champions older than Pete with no more than five applications of rules.

In the absence of any specification the limits are all taken to be INF, except for RULES, which is never allowed to exceed a limit determined by the implementation, normally 1000.

8.5 SETUP

The preceding commands are special adaptations of the basic general deduction primitive, SETUP. SETUP takes three arguments. In the command \((\text{SETUP } S X C)\) the arguments \(S\), \(X\) and \(C\) are (LISP) evaluated before the SETUP procedure is entered (SETUP is an EXP). The first argument \(S\) (the "scope indicator") is an expression which evaluates either to a nonnegative integer or else to the identifier ALL. The second argument \(X\) is an expression which evaluates to an answer template. The third argument \(C\) is an expression which evaluates to a constraint. The command \((\text{SETUP } S X C)\) returns a list of the recursive realizations of the answer template \(X\) corresponding to the solutions which satisfy the constraint \(C\) in the current knowledge base. If the value of \(S\) is ALL, then all such recursive realizations are in the list returned. If the value of \(S\) is the integer \(K\), then no more than \(K\) such recursive realizations are returned. Thus the command \((\text{ALL } (x \ y) \ (\text{Age } x \ 1928 \ y))\) is equivalent to the command

\[
(\text{SETUP} \ (\text{QUOTE ALL}) \ (\text{QUOTE } (x \ y)) \ (\text{QUOTE} \ (\text{Age } x \ 1928 \ y)))
\]

and both return the list.
as their result, if the current knowledge base contains only the assertions HERBRAND0, HERBRAND2, TURING1, TURING2 and AGE-RULE. The command

\texttt{(THE logician (born logician something February 1908))}

returns the result: Herbrand .

Recall that the answer template may be a proper name, a variable, or a list of expressions. In the first case the answer is just the answer template. If the template is a variable, each answer is the simplification of the recursive realization of the answer template in a solution environment. If the template is a list of expressions, the answer is a list of simplifications of recursive realizations of expressions in the template.

8.6 NONDETERMINACY OF DEDUCTIVE PROCESSES

The order of the items in the lists returned by ALL, ANY and SETOF is not defined, nor is there defined any rule for selecting a subset of all instances when less than all are requested.

This non-determinacy is accompanied by a measure of "concurrency", in that the order in which LISP evaluations will be performed in the course of various simplifications is also not specified. The evaluation of a single evaluable expression is, however, carried out "indivisibly". It is for this reason that assignment and other side-effect-producing operations must be used with caution in LOGIC.

8.7 CONTROLLING THE DEDUCTION PROCESS

Having emphasized the non-determinacy of the deduction process, we should now point out that the user can, in fact, exercise a considerable degree of control over it, even to the point of making it fully deterministic.

8.7.1 The Heuristic Solution Cost

Recall from chapter 2 that the node selected for further progress is always one whose heuristically estimated solution cost is least. This cost is computed as

\[ *\text{DEPTHC} \times \text{ASSERTIONS}(X, Y) + *\text{LENGTHC} \times \text{NODESIZE}(X, Y) \]

where \(*\text{DEPTHC} and *\text{LENGTHC are (global) LISP identifiers, both set initially to 1. These coefficients may, however, be set to} \]
any integer values one likes, so long as the magnitudes of the resulting costs are less than $2^{18}$.

The standard settings give a reasonable heuristic search scheme, but other settings may prove useful. If one puts $\#DEPTHC = 1$, $\#LENGTHC = 0$, for example, the resulting search is breadth first, while setting $\#DEPTHC = -1$, $\#LENGTHC = 0$ gives depth-first search. In neither of the latter cases is the order specified in which the deduction tree is explored.

### 8.7.2 The PROLOG Mode

As mentioned earlier, the user can obtain a strictly determined depth first search by placing the system in the "PROLOG" mode. This is accomplished by the command (PRULOG ON). In this mode the search is, first of all, depth first. The resolvents of a particular node will, moreover, be explored in the order in which the corresponding assertions appear in the knowledge base (this is the order in which the assertions are printed by PRINTFACTS). It is this ordering of the search that distinguishes the PROLOG mode from the depth first search produced by adjusting the solution cost coefficients. If a special rule (see Chapter 3, Section 11) is in effect for a predicate which also has assertions, the special rule is considered to come after any data and before any rules.

The heuristic search mode is selected by (PROLOG OFF). One is not allowed to change modes while a search is in progress. Any attempt to do so will be met by the response "(Not while searching)". To inquire about the current search mode, use the command: (SEARCHMODE). In heuristic mode, this returns the message (HEURISTIC dm), where d and m are the current values of $\#DEPTHC$ and $\#LENGTHC$ respectively. In PROLOG mode, the response is: DEPTH-FIRST.

### 8.8 "ONE RESOLVENT" PROCEDURES

It sometimes happens that the programmer can determine that on every call of a particular procedure at most one resolvent can lead to success. Such a determination usually depends both on the nature of the queries that can be expected and on the nature of the assertions which constitute the procedure. If it can further be arranged that this resolvent always results from the first assertion which yields a resolvent, then one may inform the system of these facts by declaring the procedure in question to have the attribute ONERES. This is done with the command (PROCEDURE Pred ONERES), "Pred" being the predicate of the procedure. If a special rule (see Chapter 3) is in effect for "Pred", the special rule is considered to come between data and
rules.

The conditions under which one may appropriately specify a procedure to be "ONERES" may seem rather restrictive, but they are not uncommon in practice. An inappropriate ONERES attribution will, of course, have a drastic effect on the meaning of a procedure, since the system will in any case compute at most one resolvent for each call, even if more than one resolvent can lead to success.

8.9 SUBSCRIPTED VARIABLES IN DEDUCTIONS

We have already mentioned that variables appearing in assertions are (implicitly) given subscripts when the assertions are used in deductions so as to avoid improper identification of variables. Variables in the query are given the subscript 0. For an unsubscripted variable, say x, the system identifies x:0 with x, so as to prevent an ugly profusion of 0 subscripts. No such identification is made for a subscripted variable such as y:2; however, which would appear in the deduction as y:2:0. When resolving an assertion with a constraint, variables in the assertion are given a subscript one greater than the largest subscript used in deducing the constraint. No new subscript is introduced when resolving with a datum, nor by the special rules for *, AND, OR and COND, which introduce no variables.

Variables in answers require a bit more discussion. If a variable from the query appears in an answer it appears in its original form, without the 0 subscript added during the deduction. If a variable from an assertion appears in an answer, the treatment depends on the nature of the query. If a primary query, that is, one invoked from LISP, the variable simply appears with the subscript given in the deduction. If a subsidiary query, that is, one invoked recursively within some larger deduction, the query must have resulted from the reduction of an expression whose variables were given a subscript i ≥ 0, while in the subsidiary deduction the variable was given a subscript j > 0. Such a variable, say x, appears in an answer (to the subsidiary query) as x:j:i. Since subscripted variables cannot appear in the knowledge base, this prevents unintended identification of variables in almost all cases of practical interest. We should point out, however, that if one assertion causes two subsidiary deductions, and the answers to both contain variables introduced in the course of these deductions, it is conceivable that the same variable might appear in answers to both queries. Even in this case, such variables must appear inside quotations, and can enter the deductive process only if they are "exposed" by means of the special construct (LOGIC ...).
At this point the whole subject may seem overwhelmingly complicated, but we remind the reader that the programmer can ordinarily ignore the matter completely, and that the implementation achieves these effects implicitly and quite economically. In particular, variable identifiers like x:3:2 are never created in the internal workings of deduction; they arise only when needed for "export" to LISP.
CHAPTER 9
MONITORING DEDUCTIONS

Provision has been made for the optional "viewing" of a deduction process as it is happening. Ideally such a facility would show the tree of constraints growing during the execution of the deduction cycle. This is however somewhat extravagant of display space, and LOGIC has a more modest version of this idea.

9.1 THE MONITOR FACILITY

Execution of the command (MONITOR ALL) enables the system to display each successive selected constraint during the deduction process. If the selected (implicit) constraint is Q(E), the display shows the (explicit) constraint Q(E). In order to give the user time to reflect, the system pauses once each cycle, and resumes on receiving a suitable input (normally, a semicolon). The predications comprising the query Q(E) are displayed as they exist before any simplification is performed. It should be noted that when viewing a developing deduction process in this way one may observe some discontinuity in the display. This is because the selection mechanism may not always choose a successor of the previously selected constraint, but rather "resume" some older constraint whose turn has arrived for some more "progress". Even when the genetic thread remains unbroken, there may be rather drastic changes in the constraint owing to the LISP-simplification step of the cycle. The user will soon become accustomed to the realities of the MONITOR display, however, and will find it an enlightening tool when sparingly used to slow down and observe the deductive action. The command (MONITOR OFF) disables the MONITOR facility.

One need not simply continue from the MONITOR pause. The commands one can give are as follows (the prompt is "?*?

?*E expr - Evaluate expr and print the result
?*EXPLAIN - Explain the current state
?*QUIT - Abandon the search
?*HELP - Print brief instructions

Any other input is taken as a command to proceed. E, EXPLAIN and HELP leave the system in the MONITOR pause. EXPLAIN may be followed by qualifiers to specify the mode of explanation (see the next chapter).

- 9-1 -
9.1.1 Controlling The MONITOR Facility

One may wish to monitor only selected steps in the deduction. To do so, one executes the command \((\text{MONITOR } P_1 \ldots P_n)\) for some predicates \(P_1 \ldots P_n\). Thereafter, the system will monitor just those cycles for which the selected constraint begins with a predication whose predicate is among \(P_1 \ldots P_n\), or for which the selected constraint is empty. We say that the predicates specified have been "flagged" for monitoring. One can flag additional predicates by executing a similar MONITOR command, or "unflag" certain predicates with a command \((\text{UNMONITOR } P_1 \ldots P_n)\). \((\text{UNMONITOR ALL})\) unflags all currently flagged predicates.

The \((\text{MONITOR OFF})\) and \((\text{MONITOR ALL})\) commands operate independently of flagged predicates, and without changing the flags. The command \((\text{MONITOR ON})\) re-establishes selective monitoring.

One may also wish to observe constraints after simplification as well as before. The command \((\text{MONITOR 2})\) causes the system to print the constraint after simplification, in addition to the normal display before simplification, provided that the constraint was altered in some way by simplification. If selective monitoring is in effect, the decision as to whether the cycle should be monitored at all is still based on the initial predicate of the selected constraint, before simplification. The command \((\text{MONITOR 1})\) restores the normal mode, printing the constraint before simplification only.

The numerals "1" and "2" can be included in MONITOR commands which flag predicates, in which case they have the same effect as when they stand alone. The key words OFF, ON and ALL are not recognized in such commands, however, so the command \((\text{MONITOR OFF Male})\) would flag the predicates OFF and Male and enable selective monitoring.

9.2 THE PURR FACILITY

It is often desirable to be able to see in some direct way that the deduction process is taking place, without necessarily slowing it down to the extent that the MONITOR facility entails. The command \((\text{PURR ALL})\) enables just such a facility, the PURR facility. The PURR facility consists of a running display accompanying the deduction process. It involves the printing of a few single characters per cycle. No line feed is given after printing (except at the physical end of a line) so that the characters form a continuous string. The meaning of each character is as follows:

- 9-2 -
### Character | Meaning
--- | ---
{l} | Start of a new query
{- (hyphen)} | Start of a new cycle
{P} | Selected constraint a success
{U} | Selected predication is NIL (false)
{R} | Resolvents of selected constraint obtained
{X} | Selected constraint failed for lack of resolvents
{C} | A continuation popped up
{L} | Selected constraint failed due to window limit
{J} | Completion of a query

The PURR facility is disabled by the command (PURR OFF). Thus with the PURR facility on the following transaction would occur:

```
*(ALL (x y) (Age x 1920 y))
[-R-R-R-P-R-P]
((Turing ε.) (Herbrand 12.))
*
```

The "PURR string" shows that the deduction took six cycles, invoked four procedures and found two answer environments. Note that if a query is invoked within the processing of another query the PURR string will contain nested bracket pairs.

#### 9.2.1 Selective PURRING

When following long deductions, two characters or so on every cycle may seem excessive, and one can specify selective PURRING in a manner closely akin to that used to specify selective monitoring. As with the monitor facility, control is based on the initial predicate of the selected constraint, and predicates are flagged for purring with commands of the form (PURR Pl...Pn), unflagged with commands of the form (UNPURR Pl...Pn). Empty constraints (successes) are always selected. The key words OFF, ON and ALL are used exactly as with MONITOR. Numerals are allowed in PURR commands, but have no effect.

PURR and MONITOR are not ordinary FEXPRs, since they will evaluate a (first) argument which is not atomic. This allows one to nest calls of these functions, as in (PURR (MONITOR ALL)), which enables both PURRING and MONITORing on all cycles.
CHAPTER 10
EXPLAINING DEDUCTIONS

Once a deduction has been completed and its answer list obtained, one may call for an explanation of the reasoning by which some or all of the answers were deduced. For instance, the following transaction consists of first constructing the answer list for the query *(ALL (x y) (Age x 1920 y))* and then requesting an explanation for the second item.

*(*(ALL (x y) (Age x 1920 y))
((Turing 8.) (Herbrand 12.)))

*(EXPLAIN 2)*

To show:
((Age x 1920. y))

it is enough, by
(1- AGE-RULE (Age x 1920. y)
  <- (Born x # # birth-year:1)
    & (= y (- 1920. birth-year:1)))

to show:
((Born x # # birth-year:1) (= y (- 1920. birth-year:1)))

then it is enough, by
(1- HERBRAND1 (born Herbrand 12. February 1908.))

to show:
((= y 12.))

then it is enough, by
(1- REFLEXIVE-LAW (= Reflexive Law))

to show:
NIL
(End of explanation)

The *(EXPLAIN 2)* command causes an explanation of the answer (Herbrand 12.) to be printed. The successive constraints leading to the answer are exhibited, and the assertion activated to cause each transition is shown. The activated assertion is shown with
respect to the environment part of the resulting constraint (i.e., after the activation has extended the environment). Various further inflections are provided with the EXPLAIN command. (EXPLAIN ALL) provides explanations of all answers. (EXPLAIN Ni ... Nk) provides explanations of the Ni-th ... Nk-th answers. (EXPLAIN) is the same as (EXPLAIN 1).

Explanations can be produced only when the history facility is enabled, which normally it is not. The history facility is enabled by (HISTORIES ON), disabled by (HISTORIES OFF). Enabling the history facility can impose significant overhead on the system, particularly when the deduction tree must be searched to great depth.

The answers which one can have explained are those produced by the most recently completed invocation of ALL, ANY, THE or SETOF. If there are no such answers EXPLAIN will simply respond "(Nothing to explain)". An attempt to select a non-existent answer will be ignored, except that a note to that effect is typed.

10.1 ALTERNATIVE EXPLANATION MODES.

The EXPLAIN facility is considerably more flexible than indicated by the example just discussed, which illustrates only the normal mode of explanation. One can obtain explanations in a variety of styles. The variations are specified by typing qualifiers in the command following the selection of the answers to be explained. To illustrate, the command (EXPLAIN 2 NAMES FINAL) would print a similar sort of explanation, except that only the names of the assertions would be printed, and the constraints would all be recursively realized in the solution environment.

10.1.1 Specifying Items To Be Included.

Besides constraints and assertions, one may also instruct the system to print answer templates at each stage of the explanation, instantiated and simplified. One may also print names of assertions rather than printing assertions in full.

When names of assertions are to be printed the system will construct names for assertions for which the user has not specified names. These "manufactured" names have the form (Pred Rule k) or (Pred Datum k), following the conventions discussed in chapter 5. User-supplied names are usually taken just as specified, but one can request "long" names, in which case the name given by the user is combined with the principal predicate symbol to form a list "(Pred Name)". Manufactured names are always in the long format.
The qualifiers which control all this are the following:

<table>
<thead>
<tr>
<th>Qualifier</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASSERTIONS</td>
<td>Print assertions in full</td>
</tr>
<tr>
<td>NAMES</td>
<td>Print names of assertions</td>
</tr>
<tr>
<td>UNNAMED</td>
<td>Print assertions which lack user-supplied names, print names where available</td>
</tr>
<tr>
<td>LONG</td>
<td>Print all names in long format</td>
</tr>
<tr>
<td>SHORT</td>
<td>Print user-supplied names in short format</td>
</tr>
<tr>
<td>CONSTRAINTS</td>
<td>Print constraints</td>
</tr>
<tr>
<td>NOCONSTRAINTS</td>
<td>Omit constraints</td>
</tr>
<tr>
<td>ANSWERS</td>
<td>Print answer templates</td>
</tr>
<tr>
<td>NOANSWERS</td>
<td>Omit answer templates</td>
</tr>
<tr>
<td>CONTINUATIONS</td>
<td>Print continuations with constraints</td>
</tr>
<tr>
<td>NOCONTINUATIONS</td>
<td>Omit continuations</td>
</tr>
</tbody>
</table>

If NOCONSTRAINTS is specified the format of the explanation is adjusted accordingly. If NOCONSTRAINTS, NOANSWERS and NAMES are all specified the explanation is simply a list of the names of the assertions used, with no ornamentation. The default selection between CONTINUATIONS and NOCONTINUATIONS can be changed by (CONTINUATIONS ON) or (CONTINUATIONS OFF).

10.1.2 Specifying Environments to be Used.

We remarked earlier that the normal explanation shows each step of the derivation in the environment current at that step. One can, however, specify other choices as follows:

<table>
<thead>
<tr>
<th>Environment</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>INITIAL</td>
<td>Use initial (empty) environment</td>
</tr>
<tr>
<td>CURRENT</td>
<td>Use current environment</td>
</tr>
<tr>
<td>FINAL</td>
<td>Use final (solution) environment</td>
</tr>
</tbody>
</table>

When the INITIAL environment is specified constraints are shown in the current environment, as nothing earlier makes any sense, while assertions are shown in the form in which they appear in the knowledge base. Note that the ANSWERS option is useful only in conjunction with CURRENT, though other combinations are allowed.

Anything other than a qualifier appearing in the command will be ignored, with a warning message to that effect typed to the user.
10.2 LIMITING EXPLANATIONS

The full explanation of an answer, as normally produced by LOGLISP, can be quite lengthy, and one might wish to limit the explanation by omitting certain uninteresting steps. If one declares the predicate 'Pred' to have the attribute NOHIST, using, say, the command (PROCEDURE Pred NOHIST) explained in chapter 4, then histories recorded by the system will omit deduction steps which used assertions from procedure Pred, and subsequent explanations will omit such steps as well.

The following example shows the effect of suppressing the step using the procedure Born in the deduction of Herbrand's age in 1920.

* (PROCEDURE Born NOHIST)
Born
*(ALL (x y) (Age x 1920 y))
((Turing 8.) (Herbrand 12.))
*(EXPLAIN 2)
To show:
((Age x 1920. y))
it is enough, by
(;- AGE-RULE (Age Herbrand 1920. y) <- (Born Herbrand ! # 1908.)
& (= y (-- 1920. 1908.)))
to show:
((= y 12.))
then it is enough, by
(;- REFLEXIVE-LAW (= Reflexive Law))
to show:
NIL
(End of explanation)

One observes that the omitted steps are not entirely ignored, since the bindings these introduce may influence the appearance of the steps which are retained in the explanation.
10.3 OBTAINING EXPLANATIONS IN LISP.

The system contains a number of SUBK-type functions which allow
the LISP programmer to get at the basic material of the
explanations. The programmer can then format explanatory
material in whatever way he finds convenient. The first argument
to each of these functions is an "answer number", which is the
number of the answer to be explained, just as might be typed to
EXPLAIN. The effect on these functions of predicates with the
NOHIST attribute is analogous to the effect on EXPLAIN.

(EXPLNAMES ANSNMB)

returns a list of the names, in long format, of the assertions
used to derive the answer, in the order used.

(EXPLASSERTIONS ANSNMB ENV)

returns a list of the assertions used to derive the answer, in
the order used. Here ENV should be one of the atoms INITIAL,
CURRENT, FINAL, to specify the environment in which the
assertions will be shown. Each assertion is represented by a
list

     (Pred Datum/Rule Name/Number Head T1 ... T1)

where "Pred" is the principal predicate symbol, "Datum/Rule" is
either the identifier "Datum" or the identifier "Rule", according
to the classification of the assertion, "Name/Number" is the
user-supplied name or system-manufactured number, and the
remaining entries are the predications of the assertion.

(EXPLCONSTRAINTS ANSNMB ENV CONTNS)

returns a list of the constraints arising in the derivation,
beginning with the original query and ending with NIL. Here ENV
specifies the environment as before, except that INITIAL is
treated the same as CURRENT. CONTNS should be T if continuations
are desired, NIL otherwise. The entries of the list returned by
EXPLCONSTRAINTS are themselves lists of some complexity. If the
constraint in question has no continuation, the corresponding
entry has the form:

     (q1 ... qN))

where qi is a predication. If the constraint has a continuation,
but CONTNS is NIL, the entry will have the form

((ql ... qN) CONTINUED)

while if the constraint has a continuation and CONTNS is T, the entry has the form

((ql ... qN) (pl ... pM) ...)

where pl is a predication of the continuation, which may itself be followed by another continuation, and so on.

(EXPLTEMPLATES ANSNUMB)

returns a list of answer templates shown in the successive CURRENT environments, beginning with the original template and ending with the actual answer.

All of these functions follow a common convention regarding exceptions. If the answer number specified does not correspond to an existing derivation the result is the atom NO-EXPLANATION. If the most recent search was performed with the history facility disabled the result is NIL.
CHAPTER 11
INTERACTING WITH LOG LISP

In the present chapter we discuss the mechanics of running LOG LISP, obtaining information, controlling the operating modes and default settings, and some points dealing with errors. Before doing so we emphasize one convention:

RESERVED IDENTIFIERS

Identifiers beginning with the character "#" are reserved for use by the system. Users should generally avoid such identifiers. Under no circumstances should a user assign a value to such an identifier.

11.1 RUNNING LOG LISP

We suppose that the user has logged in and obtained access to the disk area containing the LOG LISP system. The precise method for doing so will vary from one installation to another.

To run the LOG LISP system simply type the monitor "command"

./LOG LISP core

where "core" is an optional core argument in the form one would give for the RUN command. If the core argument is omitted the system will have a rather small working area. A good medium allocation is 90K. The maximum core allocation is 164K, which results in a low segment of just over 128K. Core requests outside the usable range are adjusted to the nearest allowable value. For large programs one may wish to specify the LISP storage allocations. To do so, use a command like

./LOG LISP 140,10000 1000 1000 1000

in which the core argument is followed by a comma and the LISP allocations, separated by spaces. The order of these is FULL...
WORD SPACE, BINARY PROGRAM SPACE, REGULAR PDL, SPECIAL PDL, with
the allocations being interpreted in octal, just as in LISP.

LUGLISP will print a version message and prompt with "*" when
ready, being at the top level of LISP. At this point one may
enter assertions, queries, and the like as described in the
earlier chapters. The system includes the optional numeric
functions (SQRT, SIN, SIND, etc.) loaded in binary program space.

The commands just described depend on the availability of the MIC
(Macro Interpreted Command) system. At installations not
providing MIC one must use the alternate method described in the
next section. Some installations running MIC do not provide the
compact "/" commands, in which case the longer form

.DO LUGLSP core

may be used (with allocations if desired).

11.1.1 An Alternate Method

The method of running LUGLISP to be described now is useful only
for those who desire non-standard initialization or wish to
minimize the core requirements of the system. One may also run
the system using the monitor command

.RUN LUGLSP core

in which case LISP will ask for allocations. The system will be
run uninitialized with no version message, but still including
the extra numeric functions. Before assertions can be entered or
queries processed the system must be initialized using START (see
below).

11.2 INITIALIZATION

When run in the usual way the system starts out properly
initialized with an empty knowledge base. One may re-initialize
the LOGIC part of the system at any time by invoking the function

(START)

leaves an empty knowledge base and resets the operating mode
controls and system defaults to their standard values. LISP
function definitions, file descriptions, and identifier values
are not changed, except for those values which are used in system
control.
11.3 INFORMATION

When prompted for input at any of the main interaction points the user can obtain brief instructions by simply typing HELP. Assistance is thus available at the top level of LISP, in the monitor pause, and in FACTS, as well as when the deduction machinery asks for instructions (see below). HELP is not available while editing, but the editor is just the standard LISP editor, so no special difficulties should be encountered.

Abbreviated instructions for using any of the LOGIC interface functions (\texttt{!-}, \texttt{THE}, \texttt{ALL}, \texttt{ANY}, etc.) can be obtained by invoking the command \texttt{(DOC \textit{fn})}, where \textit{"fn"} is the name of the function in question. These instructions were developed using the on-line documentation package described in appendix A. The documentation package itself is included in LOGLISP for the convenience of users.

11.4 CONTROL

The earlier chapters of this report mention a number of functions used to control various operating modes, as well as several defaults used by the system. In this section we shall summarize the control functions and explain the treatment of defaults in somewhat greater detail.

11.4.1 Control Functions

With the exception of \texttt{PURR} and \texttt{MONITOR} (see Chapter 9), all of the control functions take one argument, which should be \texttt{ON} or \texttt{OFF} (\texttt{T} or \texttt{NIL} may be used as well), and return the argument after altering the system state appropriately. These functions will, however, evaluate a non-atomic argument expression, so that calls of the functions may be nested. To illustrate, the command \texttt{(HISTORIES \textbf{CONTINUATIONS ON})} enables both the recording of \texttt{HISTORIES} and the printing of \texttt{CONTINUATIONS}.

Several of these functions operate simply by setting the value of a LISP identifier, in which case \texttt{NIL} represents \texttt{OFF}, while anything else represents \texttt{ON}. (\texttt{PURR} and \texttt{MONITOR} use \texttt{ALL} to represent the state selected by \texttt{ALL}.) The identifiers so used may be changed directly by LISP programs, or accessed by them as may seem useful. The table which follows lists the names of the control functions, the initial settings, and, where applicable, the identifier set by the function.
<table>
<thead>
<tr>
<th>Function</th>
<th>Initial Setting</th>
<th>Identifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>PURR</td>
<td>OFF</td>
<td>*PURR</td>
</tr>
<tr>
<td>MONITOR</td>
<td>OFF</td>
<td>*MONITOR</td>
</tr>
<tr>
<td>CONTINUATIONS</td>
<td>OFF</td>
<td>*CONTINUATIONS</td>
</tr>
<tr>
<td>HISTORIES</td>
<td>OFF</td>
<td>*HISTORIES</td>
</tr>
<tr>
<td>ASK</td>
<td>ON</td>
<td>*ASK</td>
</tr>
<tr>
<td>AUTO=</td>
<td>ON</td>
<td>[None]</td>
</tr>
<tr>
<td>AUTOAND</td>
<td>ON</td>
<td>[None]</td>
</tr>
<tr>
<td>AUTO-OR</td>
<td>ON</td>
<td>[None]</td>
</tr>
<tr>
<td>AUTOCOND</td>
<td>ON</td>
<td>[None]</td>
</tr>
<tr>
<td>PROLOG</td>
<td>OFF</td>
<td>[None]</td>
</tr>
</tbody>
</table>

Initial settings are reestablished by START. The facility controlled by ASK is described below in the discussion of errors.

11.4.2 Defaults

Both in specifying deduction windows and in requesting explanations the user normally relies on many defaults. These are not, in fact, determined rigidly by the system, but may be adjusted by the user. The standard default settings are, however, restored by (START).

11.4.2.1 Deduction Window Defaults - The defaults for deduction windows are the values of the LISP identifiers listed below, along with their initial values.

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>*TREESIZE</td>
<td>INF</td>
</tr>
<tr>
<td>*NODESIZE</td>
<td>INF</td>
</tr>
<tr>
<td>*ASSERTIONS</td>
<td>INF</td>
</tr>
<tr>
<td>*RULES</td>
<td>1000</td>
</tr>
<tr>
<td>*DATA</td>
<td>INF</td>
</tr>
</tbody>
</table>

Each of these gives the default value for the corresponding window limit. Though 1000 is the normal value for *RULES, a different initial value will be used if the system was specially constructed. The implementation constraint on the number of rules in a single deduction will be rigorously enforced, even if *RULES is made larger than this limit.

The values which one may assign to these identifiers are the atom INF or any non-negative integer.
11.4.2.2 EXPLAIN Defaults - The default qualifiers for EXPLAIN are similarly controlled by a collection of LISP identifiers. The table below shows the identifiers, the set of values each is allowed to take, and the initial value.

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Value Set</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>*ASSERTIONS</td>
<td>(ALL, SOME, NIL)</td>
<td>ALL</td>
</tr>
<tr>
<td>*CONSTRAINTS</td>
<td>(T, NIL)</td>
<td>T</td>
</tr>
<tr>
<td>*LONGNAMES</td>
<td>(T, NIL)</td>
<td>NIL</td>
</tr>
<tr>
<td>*ANSWERS</td>
<td>(T, NIL)</td>
<td>NIL</td>
</tr>
<tr>
<td>*CONTINUATIONS</td>
<td>(T, NIL)</td>
<td>NIL</td>
</tr>
<tr>
<td>*ENVIRONMENT</td>
<td>(FINAL, CURRENT, INITIAL)</td>
<td>CURRENT</td>
</tr>
</tbody>
</table>

Note that *CONTINUATIONS is controlled by the function CONTINUATIONS, and affects the monitoring facility as well as EXPLAIN.

11.5 ERRORS

Errors can arise either in LOGIC or in LISP.

11.5.1 LISP Errors

Errors detected by LISP will result in entry to the LISP break package in the usual way. If the error arose during simplification a backtrace will show none of the workings of the reduction machinery, which is probably the best course the system could take.

All of the LISP facilities for recovery and analysis are available. There are, in addition, two special break commands which may be helpful.

LOGBK - Prints a "logic backtrace" showing the expression being simplified and the constraint from which it arose.

EXPLAIN - Prints a standard explanation of the constraint being simplified, provided histories are being recorded. EXPLAIN may be followed by qualifiers, as in "EXPLAIN NAMES".

Note that misspelled function names in LOGIC terms will not lead to undefined function errors, simply to expressions which are not evaluable.
11.5.2 LOGIC Errors

Earlier chapters explained how syntax errors are handled by LISP and FACTS. There is one other type of error which can be detected by LOGIC -- the "undefined predicate" error.

A predicate is considered to be undefined if it has neither a LISP definition (as a function) nor a LOGIC definition (as a procedure of one or more assertions). If such a predicate is encountered during a search, and if the ASK facility is enabled (as it is initially), the system will ask the user for instructions, after first printing a message specifying the undefined predicate.

The prompt for instructions is "ASK*". Responses are as follows:

- **ASK;** Continue search
- **ASK** Execute FACTS
- **ASK*S** Correct spelling automatically, if possible
- **ASK*S pred** Correct spelling to pred
- **ASK*E expr** Evaluate expr and print the result
- **ASK*P** Print the current constraints (as when monitoring)
- **ASK*QUIT** Abandon the search
- **ASK*HELP** Print instructions

Anything other than ";" causes the system to remain in the ASK state. If the user does anything which might conceivably alter matters, the system will try again to simplify and obtain resolvents.

The automatic spelling correction attempts to find a predicate (defined by LOGIC) which closely matches the undefined predicate. If successful it informs the user of the chosen predicate, if not successful it informs the user of that fact. Spelling corrections are accomplished with REPLACA, so the effect may reach beyond the immediate situation. When the undefined predicate occurs as an instance of some variable, spelling corrections are probably unwise, and the user is warned of such circumstances.

11.5.3 WAITING Limit Exceeded

If the limit on WAITING nodes is reached and a new node needs to be entered, the system will discard the new node and print a message to that effect. No provision is made for user intervention upon such an occurrence.
11.5.4 Exhaustion Of Free Storage

If a deduction is terminated because of exhaustion of free storage (signified by the message NO FREE STG LEFT) there may be many nodes WAITING to be processed, and there may not be enough storage to start another search until these nodes are erased. To do so, one may give the command (-INITHEAP 1). This invokes an internal function which accomplishes the desired result, setting the search mode to (HEURISTIC 1 1) as it does so. Afterwards it may help to run with (HISTORIES OFF).

11.6 ADDITIONAL LISP FUNCTIONS

LOGLISP includes a number of functions not provided by standard LISP. Some of these have been mentioned earlier.

11.6.1 Short Names For Arithmetic

The short arithmetic operators are as follows:

\begin{align*}
(+ & e_1 \ldots e_N) \quad \text{[MACRO]} \\
(- & e_1 \ldots e_N) \quad \text{[MACRO]} \\
(* & e_1 \ldots e_N) \quad \text{[MACRO]} \\
(\% & e_1 \ldots e_N) \quad \text{[MACRO]}
\end{align*}

These are the same as PLUS, DIFFERENCE, TIMES, QUOTIENT, except for being more defined. \((+) = (-) = 0,\) while \((\ast) = (\%) = 1.\)

11.6.2 Arithmetic Relations

The following arithmetic relations are provided, in addition to those included in LISP:

\begin{align*}
(< & e_1 e_2) \quad \text{[SUBR]} \\
(<= & e_1 e_2) \quad \text{[SUBR]} \\
(>= & e_1 e_2) \quad \text{[SUBR]} \\
(> & e_1 e_2) \quad \text{[SUBR]}
\end{align*}

Of course "=" is defined on numbers as well as other objects.
11.6.3 Miscellaneous Arithmetic

Two other special arithmetic functions are provided:

\[
\text{** N) [SUBR]}
\]
returns \(x \times N\) for integer \(N\).

\[
\text{ODD N) [SUBR]}
\]
returns \(T\) if the integer \(N\) is odd, \(NIL\) otherwise.

11.6.4 LUGLISP Utilities

Some of the LUGLISP system utility functions may be of use to programmers. The names of these functions are not reserved.

\[
\text{(VARIABLE e) [SUBR]}
\]
returns \(T\) if \(e\) is a LOGIC variable, \(NIL\) otherwise. In an assertion one might write \((\text{VARIABLE (LISP x)})\) to determine whether the instantiation of \(x\) is or is not a variable.

\[
\text{(CONS M e1 ... eN-1 eN) [MACRO]}
\]
returns the object \((v1 ... vN-1 vN)\), where \(v1\) denotes the value of \(e1\).

\[
\text{(XFERPROP "DST" SRC "KEY") [FSUBR]}
\]
makes property \(KEY\) of \(SRC\) also be property \(KEY\) of \(DST\). The property value is not copied.

\[
\text{(Version) [SUBR]}
\]
prints a message identifying the version of LUGLISP in use.

\[
\text{(HELP) [SUBR]}
\]
prints a classified list of logic system functions.
11.6.5 Date And Time Functions

(DATEM) [SUBR]
returns the date as a list of the form (day month year), where "day" and "year" are integers and "month" is one of (Jan Feb Mar Apr May June July Aug Sept Oct Nov Dec).

(TIMER) [SUBR]
returns the time of day as a list of the form (hr : min), where "hr" and "min" are integers.

(DAYTIME) [SUBR]
returns the date and time as a list of the form (day month year hr : min).
Applications of logic programming are described by
[Kowalski 1979], [Clark 1979], [van Emde Boas 1977],
[Colmerauer 1973], and [Warren 1977], to name only the principal
references.

In this chapter we describe two non-trivial examples of logic
programming in which the special features of LOGLISP are
exploited.

12.1 PLACES - AN "INTELLIGENT" DATABASE.

Logic programming lends itself naturally to the creation and
operation of "intelligent" databases. Such databases are capable
of being "told" facts and rules and of being "asked" questions
whose answers in general may require reasoning.

The idea is to design the database so that both telling and
asking can be done in a reasonably free style which does not
require conformity to pre-designed formats. Ideally both telling
and asking would be done in ordinary informal natural language -
but this is at present a major problem. An approximation to such
a system can be achieved in LOGLISP by using (formal) assertions
to tell and (formal) queries to ask. As an example of an
"intelligent" database we put together a LOGLISP knowledge base
called PLACES. PLACES contains several thousand assertions -
most of which are data, i.e., unconditional ground assertions.

Some representative data of PLACES are shown in Figure 1. For
each predicate appearing in Figure 1, PLACES has a collection of
such unconditional ground assertions - a data procedure. All
these data procedures are comprehensive (they average several
hundred assertions each) and some are in a sense incomplete.
Resolving a constraint \((\mathbf{Q}, \mathbf{E})\) against a large data procedure \(\mathbf{P}\) does not in general require that each of \(\mathbf{P}\)'s many assertions be checked to see if its header will unify in \(\mathbf{E}\) with the selected predication \(\mathbf{A}\). The secondary indexing scheme of LOGLISP allows the efficient retrieval of just those assertions whose headers contain one of the proper identifiers which occur in \(\mathbf{A}(\mathbf{E})\). In general the smallest such subset \(\mathbf{S}\) of \(\mathbf{P}\)'s assertions will be much smaller than \(\mathbf{P}\) itself, and the resolution process searches \(\mathbf{S}\) rather than \(\mathbf{P}\).

The procedures \text{POPULATION}, \text{AREA}, \text{REGION}, \text{GNP-PER-CAPITA} are complete in the sense that every country in the world is covered.

The \text{GNP-PER-CAPITA} procedure gives (in US dollars) the gnp-per-capita for each country in the world for a particular year (1976).

The procedure \text{ADJOINS} provides data for a procedure \text{BORDERS}, which is a pair of rules:

\[
\text{(BORDERS} \ x \ y) \leftarrow (\text{ADJOINS} \ x \ y) \\
\text{(BORDERS} \ y \ x) \leftarrow (\text{ADJOINS} \ y \ x)
\]

which give \text{PLACES} the ability to determine which countries (or bodies of open water) border upon which others. Since \text{ADJOINS} is a symmetric relation we need not assert it in both directions, and \text{BORDERS} uses \text{ADJOINS} accordingly.

The procedure \text{PRODUCES} gives (in millions of metric tons) the quantities of various basic commodities (oil, steel, wheat, rice) produced by most of the world's countries in two particular years (1970 and 1975). This procedure could well have covered more years and more commodities, but for the purposes of an example a
few hundred assertions seemed enough to illustrate the possibilities.

While the countries of the world form (at any given time) a rather definite set, it is less clear what are the bodies of water which should be named and treated as entities in a database such as PLACES. We took the arbitrary course of naming those bodies of water found on the maps of various parts of the world in the Rand McNally Cosmopolitan World Atlas. We ignored those bodies of water which seemed too small to be of much significance but we strove for some sort of comprehensive description of the boundary of each country. For example, the query

\[(\text{ALL} \times (\text{BORDERS} \times \text{IRAN}))\]

gets the answer

\[(\text{STRAITS-OF-HORMUZ} \ \text{GULF-OF-OMAN} \ \text{TURKEY} \ \text{USSR} \ \text{PAKISTAN} \ \text{IRAQ} \ \text{CASPIAN-SEA} \ \text{AFGHANISTAN} \ \text{PERSIAN-GULF})\]

in which each of the bodies of water STRAITS-OF-HORMUZ, GULF-OF-OMAN, CASPIAN-SEA and PERSIAN-GULF is listed as having a portion of its boundary in common with that of the country IRAN.

12.1.1 RULES.

PLACES contains, in addition to these large "data procedures", a number of rules defining predicates useful in formulating queries.

For example, there is a procedure DISTANCE, which consists of the following four rules:

- 12-3 -
This procedure can be used to obtain the great-circle distance between any two cities whose latitudes and longitudes are in the data tables, or between one such city and an arbitrary position on the earth's surface (given by its latitude and longitude) or between two such arbitrary positions.

The procedure DISTANCE illustrates the ability to call user-defined LISP functions by forming constructions using their names as operators. The LISP function SPHOST returns the great circle distance (in nautical miles) between any two points on the earth's surface (given by their respective latitudes and longitudes).

Thus the query:

\((\text{THE } d \text{ (DISTANCE (PLACE SAN-FRANCISCO) (PLACE OSLO). d)})\)

gets the answer:

5197.5394

- 12-4 -
There is a rule which serves to define the predicate LANDLOCKED. Intuitively, a country or body of water is landlocked if it borders upon only land. The PLACES rule which formalizes this meaning is

\[
\text{LANDLOCKED} \ x \ \leftarrow \ \text{IS-COUNTRY} \ x \ \\
\& \ \text{NULL} \ (\text{ANY} \ 1 \ T \ (\text{BORDERS} \ x \ z) \ (\text{OPEN-WATER} \ z))
\]

This rule contains two features worthy of comment.

The predicate IS-COUNTRY, defined by the rule

\[
\text{IS-COUNTRY} \ x \ \leftarrow \ \text{COND} \ ((\text{VARIABLE} \ (\text{LISP} \ x)) \ (\text{COUNTRY} \ y \ x)) \ \\
((\text{ANY} \ 1 \ T \ (\text{COUNTRY} \ z \ x)))
\]

shows how one can use to advantage the LISP conditional form within a LOGIC predication. The effect of the conditional is to avoid redundancy in proving that a given country is a country — by finding all the various cities in it — via a check to see if the argument \( x \) is a variable or not. If it is not, then we need find only one datum from the COUNTRY data procedure which has the given country as its second argument.

The second thing worth noting about the rule for LANDLOCKED is the embedded deduction. The list returned by the call

\[
(\text{ANY} \ 1 \ T \ (\text{BORDERS} \ x \ z) \ (\text{OPEN-WATER} \ z))
\]

will be empty if and only if \( x \) is landlocked.

A similarly structured rule defines the predicate DOMINATES. We wish to say that a country \( x \) dominates a "narrow" waterway \( y \) if \( x \) borders \( y \) but no other country does. Thus:

\[
\text{DOMINATES} \ x \ y \ \leftarrow \ \text{NARROW} \ y \ \\
\& \ \text{IS-COUNTRY} \ x \ \\
\& \ \text{BORDERS} \ x \ y \ \\
\& \ \text{NULL} \ (\text{ANY} \ 1 \ T \ (\text{BORDERS} \ y \ w) \ \\
(\text{NOT} \ (\text{OPEN-WATER} \ w)) \ \\
(\text{NOT} \ (= \ x \ w)))
\]
12.1.2 NEGATION AS FAILURE.

The use of the predicate NOT in the procedure DOMINATES raises an interesting general point.

NOT is of course a LISP-defined notion and will therefore receive appropriate treatment during the deduction cycle in the manner explained in Chapter 3.

However, it is possible to include in one's knowledge base the rule

\[(\text{NOT } p) \leftarrow (\text{NULL } \{\text{ANY } 1 \text{ T } p\})\]

which is known as the "negation as failure" rule. PLACES has the negation as failure rule as one of its assertions. The effect of its presence in a knowledge base is to declare that the knowledge base is complete - that inability to deduce \( p \) is to be treated as tantamount to the ability to deduce the negation of \( p \).

The version of the negation as failure rule shown above is undiscriminating as between the various predications - it is in effect the declaration that all of the data procedures are complete and that all of the general procedures are "definitions" of their predicates. It would be possible to assert more specialised negation as failure rules, which declare that the knowledge base is complete with respect to a particular predication-pattern. For example, we might assert

\[(\text{NOT } (\text{BELONGS } x \ y)) \leftarrow (\text{NULL } \{\text{ANY } 1 \text{ T } (\text{BELONGS } x \ y)\})\]

in order to declare that \( \text{BELONGS} \) is complete, even though we are not willing to assert the negation as failure rule for all predications \( p \). In general, one would expect that users of LOGLISP would wish to be selective in their appeal to negation as failure, in just this fashion. These data and rules are invoked by the following queries, which illustrate some of the possibilities.

12.1.3 Some Sample Queries For PLACES.

The following examples consist of some specimen queries which one can make of PLACES, together with the answers that they get. In each case we first state the query in ordinary English, and then restate it in formal LOGLISP.

We are not claiming that there is a uniform procedure, known to us, by which one may translate queries from English to LOGLISP in this manner. At present, in order to express queries land
indeed, assertions) in LOGLISP, one must know the language and be able to express one's intentions in it. In this respect LOGLISP is like any other programming language. It is in fact quite easy to learn enough LOGLISP to construct and operate one's own database in the style of PLACES.

Query 1.

What are the oil production figures for the non-Arab OPEC countries in the year 1975?

(ALL (x y)
   (BELONGS x OPEC)
   (NOT (BELONGS x ARAB-LEAGUE))
   (PRODUCES x OIL y 1975.))

Answer 1.

(((IRAN 267.59999) (NIGERIA 88.399991)
   (VENEZUELA 122.19999)
   (INDONESIA 64.100000)
   (ECUADOR 8.2000000))

This answer is shown just as the LISP "prettyprint" command SPRINT types it out. It is of course possible to dress up one's output in any way one pleases. Note that ALL returns a list of (in this case) tuples.

Query 2.

Of all the countries which are poorer than Turkey, which two produced the most steel in the year 1975? How much steel was that? What are the populations of those countries?
(FIRST 2.
(QUICKSORT
(ALL (x y w)
(GNP-PER-CAPITA TURKEY v)
(GNP-PER-CAPITA x u)
(LESSP u v)
(PRODUCES x STEEL y 1975.)
(POPULATION x w))
(DECREASING)
2.))

Answer 2.
((CHINA 27.0 880000000.) (INDIA 7.8999999 643000000.))

This example illustrates the fact that ALL (like ANY, THE, and SETUP) returns a LISP data-object which can be handed as an argument to a LISP function. In this case QUICKSORT and FIRST are user-defined LISP functions which were created in order to serve as useful tools in posing inquiries to PLACES.

(QUICKSORT list relation k) returns the given list of tuples ordered on the kth component with respect to the given relation. (FIRST n list) returns the list of the first n components of the given list. (DECREASING) returns the LISP relation GREATERP (and we also have (INCREASING), which returns the relation LESSP, and (ALPHABETICALLY), which returns the relation LEXORDER).

Query 3.

which of France's neighbors produced most wheat (in metric tons) per capita in the year 1975? How much wheat per capita was that?

(EARLIEST
(ALL (x y)
(BORDERS x FRANCE)
(PRODUCES x WHEAT z 1975.)
(POPULATION x u)
(= y (QUOTIENT (TIMES z 1000000.) u)))
(DECREASING)
2.)

- 12-8 -
Answer 3.

(ITALY 0.16956329)

(EARLIEST list relation k) returns the first tuple in list after it has been re-ordered on the kth component of each of its tuples with respect to the given relation. Note that arithmetical terms formed with LISP's arithmetic operations are evaluated by the simplification step of the deduction cycle, as explained in Chapter 3.

Query 4.

Which of the NATO countries is landlocked?

(ALL x (BELONGS x NATO) (LANDLOCKED x))

Answer 4.

(LUXEMBOURG)

Query 5.

Which waterway is dominated by Panama?

(THE x (DOMINATES PANAMA x))

Answer 5.

PANAMA-CANAL

Note that THE returns PANAMA-CANAL and not (PANAMA-CANAL).

Query 6.

Describe the boundary of the USSK by giving
all its neighbors in alphabetical order.

(Order (All x (Borders x USSR)) (Alphabetically))

Answer 6.

(Afghanistan Arctic-Ocean Baltic-Sea Bering-Sea Black-Sea Bulgaria China Finland Hungary Iran Mongolia Norway Poland Romania Turkey)

(Order list relation) returns the given list after ordering it with respect to the given relation.

Query 7.

Are there any landlocked countries in the Far East? If so, give an example.

(Any 1. x (Region x Far-East) (Landlocked x))

Answer 7.

(Mongolia)

Query 8.

Is there an African country which dominates an international waterway? Which country? Which waterway?

(Any 1. (x y) (Region x Africa) (dominates x y))

Answer 8.

( (Egypt Suez-Canal))
Query 9.

What is the average distance from London of cities in countries which have a Mediterranean coastline and which are no more densely populated than Ireland? List those countries, together with their population densities, from least crowded to most crowded.

(PROGN (SETQ COUNTRIES-AND-DENSITIES
       (QUICKSORT
        (ALL (x x-density)
        (POPULATION IRELAND irish-population)
        (AREA IRELAND irish-area)
        (= irish-density
         (% irish-population irish-area))
        (BORDERS x MEDITERRANEAN-SEA)
        (NUT (OPEN-WATER x))
        (POPULATION x x-population)
        (AREA x x-area)
        (= x-density (% x-population x-area))
        (NUT (> x-density irish-density)))
       (INCREASING)
       2))
       (SETQ AVERAGE-DISTANCE
       (AVERAGE
        (ALL distance
        (MEMBER pair
         (EVAL COUNTRIES-AND-DENSITIES))
        (= country (CAK pair))
        (COUNTRY city country)
        (DISTANCE (PLACE city)
         (PLACE LONDON)
         distance)))
       (GIVE AVERAGE-DISTANCE)
       (GIVE COUNTRIES-AND-DENSITIES)
       (QUOTE *))

Answer 9.

AVERAGE-DISTANCE is

1491.1892

COUNTRIES-AND-DENSITIES is
This example shows at somewhat more length what a LISP programmer might make of an inquiry which calls for a more involved investigation. Assignment to the LISP variable COUNTRIES-AND-DENSITIES of the answer to one LOGIC call for later use within another (as well as for output) illustrates one more way in which the LOGLISP programmer can fruitfully exploit the interface between LOGIC and LISP. GIVE is just a dressed-up PRINT command which not only prints the value of its argument expression but also prints the expression.

12.2 A COMPILER.

We shall now present a compiler for a subset of PASCAL. The compiler parses the "source" program, checks types, and generates "object" code which can be executed by LISP (with a few "run-time" utility functions). In order to keep the example small we have confined ourselves to a few statement forms, provided only the types INTEGER and BOOLEAN, with no data structures, and made no provision for declarations, simply incorporating a handful of variable identifiers directly into the language. There are no procedures, no functions, no labels, no jumps. Expressions are treated rather fully, however, given the other limitations.

Even though the language is quite limited, we feel that the example is sufficient to show that we can easily write compilers which, though slow, are entirely adequate for experiments in language design. We point out that the compiler is readily modified to produce an abstract representation of the program, rather than an executable form, as could be used for program analysis or verification.

12.2.1 Organization Of The Compiler.

The "source" program will be represented as a list of tokens, which are simply LISP atoms denoting reserved words, identifiers, constants, operator symbols, and the like. An example is

\[
\text{(BEGIN } K := K - 1 ; Y := Y \times Z \text{ END)}
\]

which will be QUOTEd when it appears as an expression in LOGIC.
Such lists read nicely enough, and the lexical analyzer required to produce such a list from a character string or text file is easily written.

Corresponding to each syntactic category (nonterminal) of the language we introduce a relation which "compiles" phrases of that category. For example, the relation for the category <statement> has the form (STATEMENT tokens rep rest), where 'tokens' is a list of tokens, as above, 'rep' is the object representation of the statement which begins 'tokens', if there is one, and 'rest' is the token list obtained by removing the initial statement from 'tokens'. We do recursive descent parsing, working from left to right without backtracking.

In some cases we wish to "parametrize" categories. The relation for <expression>, for example, has the form (EXPRESSION type tokens rep rest), 'type' being the result type of the expression. On some calls the procedure will be used to discover the type of the expression which begins 'tokens', while on others it will check that the expression in question has the proper type.

To see how this works, consider the assertion for the WHILE statement, which is

\[
(\text{STATEMENT (CONS WHILE \texttt{ti})})\]

\[
(\text{PROG NIL LOOP: (COND (I s (GO LOOP:)))})\]

\[
\text{PARSE \texttt{ti} ((EXPRESSION BOOLEAN I) DO (STATEMENT s)) c})
\]

The rule applies only to non-empty lists which begin with WHILE. Using CONS expressions to unify with lists in this fashion we avoid explicit tests for empty lists, but there is no possibility that we will attempt to take the CAR or CDR of an atom. The "object" representation is a PROG construct incorporating the components of the WHILE in an obvious way.

Recall that the syntax for the WHILE statement is

\[
\text{WHILE <Boolean expression> DO <statement> },
\]

Although we could express this directly in terms of EXPRESSION and STATEMENT, it is more convenient to use the auxiliary relation PARSE. PARSE has the form (PARSE tokens items rest). The arguments 'tokens' and 'rest' are used as before, but 'items' is a list of expressions (itself an expression) which defines a sequence of items to be parsed. Each item has one of the forms: token, (syncat var), (syncat parm var). An item of the form 'token' simply specifies that the indicated token should be
found. The form (syncat var) specifies that a phrase of the category 'syncat' should be found, its representation to be denoted by 'var'. The form (syncat parm var) is similar, except that 'syncat' is to be parameterized with 'parm'.

The assertions defining PARSE are

\[
\text{(PARSE } \text{NIL } \text{x})
\]

\[
\text{PARSE } \text{x } \text{(hd , tl) c}
\]

\[
\text{COND (ATOM (LISP hd)) (= x (CONS hd tx)))}
\]

\[
\text{PARSE (tl tl c))}
\]

Observe the use of the variable "tl" to deal with the unpredictable expression 'entries'. The expression (ATOM (LISP hd)) is always evaluable, having the value T just when 'hd' is a token.

12.2.2 The Compiler.

At this point we shall list the compiler, including the interactive documentation which has been provided for the logic procedures. Following the listing we remark further upon the techniques used. The variables "built in" to the compiler correspond to the declarations

\[
\text{DEFPROP STATEMENT ((STATEMENT tokens rep rest)) DESC)}
\]

\[
\text{(PROCEDURE STATEMENT ONRES)}
\]

\[
\text{(PARSE (STATEMENT (CONS IF tl) (COND (1 sl) , sl c))}
\]

\[
\text{PARSE (tl ((EXPRESSION BOOLEAN t) THEN (STATEMENT sl))))}
\]

\[
\text{PARSE (tl ((EXPRESSION ELSE txx))}
\]

\[
\text{PARSE (STATEMENT txx s2 c) (= s ((s2))))}
\]

\[
\text{PARSE (STATEMENT WHILE tl) (PROG NIL LOOP: (COND (1 s (GO LOOP:))))}
\]

\[
\text{PARSE (tl ((EXPRESSION BOOLEAN t) DO (STATEMENT s))))}
\]

- 12-14 -
(:- (STATEMENT (CONS BEGIN t1) (PROGN ss) c)
   <- (PARSE t1 ((REPEATE (STATEMENT ;) ss) END) c))

(:- (STATEMENT (CONS v tx) (:= v e) c)
   <- (VAR-IDENT ty v)
   & (PARSE tx (:= (EXPRESSION ty e)) c))

(DEFPROP EXPRESSION
 (EXPRESSSION type tokens rep rest))
DOC)

(PROCEDURE EXPRESSION)

(:- (EXPRESSSION ty x r c)
   <- (SIMPLE-EXPR ty x se cc)
   & (COND ((REL-OPR ty1 x se cc)
        (AND (SIMPLE-EXPR ty1 ccc se2 c)
             (= ty BOOLEAN)
             (= r (rel se se2))))
      (T (AND (= ty ty1) (= r se) (= c cc))))))

(DEFPROP REL-OPR
 (REL-OPR arg-type tokens rep rest))
DOC)

(PROCEDURE REL-OPR UNERES)

(:- (REL-OPR INTEGER (CONS r c) r c)
   <- (MEMQ r (QUOTE (< <= >>= >>= <>)))

(:- (REL-OPR BOOLEAN (CONS r c) fr c)
   <- (= fr
        (SELECTQ r
         (< y<)
         (<= y<>)
         (= =)
         (>= d>=)
         (> b>)
         (<> <>)
         NIL))

   & (NEQ fr NIL))

(DEFPROP SIMPLE-EXPR
 (SIMPLE-EXPR type tokens rep rest))
DOC)
(PROCEDURE SIMPLE-EXPR UNERES)

(!- (SIMPLE-EXPR INTEGER (CONS + t1) r c)
 <- (TERM INTEGER t1 trm cc)
  & (SIMPLE-TAIL (INTEGER trm) cc r c))

(!- (SIMPLE-EXPR INTEGER (CONS - t1) r c)
 <- (TERM INTEGER t1 trm cc)
  & (SIMPLE-TAIL (INTEGER (MINUS trm)) cc r c))

(!- (SIMPLE-EXPR ty x r c)
 <- (TERM ty x trm cc)
  & (SIMPLE-TAIL (ty trm) cc r c))

(DEFPROP SIMPLE-TAIL
 ((SIMPLE-TAIL (type prev) tokens rep rest))
DOC)

(PROCEDURE SIMPLE-TAIL)

(!- (SIMPLE-TAIL (ty u) x r c)
 <- (COND ((ADD-OPR ty x opr cc)
 (AND (TERM ty cc trm ccc)
  (SIMPLE-TAIL (ty (opr u trm)) ccc r c)))
 ((= r u) (= c x)))))

(DEFPROP ADD-OPR
 ((ADD-OPR type tokens rep rest))
DOC)

(PROCEDURE ADD-OPR UNERES)

(!- (ADD-OPR INTEGER (CONS + c) + c))

(!- (ADD-OPR INTEGER (CONS - c) - c))

(!- (ADD-OPR BOOLEAN (CONS OR c) OR c))

(DEFPROP TERM
 ((TERM type tokens rep rest))
DOC)

(PROCEDURE TERM)

(!- (TERM ty x r c) <- (FACTOR ty x f cc)
  & (TERM-TAIL (ty f) cc r c))

- 12-10 -
(DEFPROP TERM-TAIL
  ((TERM-TAIL (type prev) tokens rep rest))
  DOC)

(PROCEDURE TERM-TAIL)
(\- (TERM-TAIL (ty u) x r c)
  \- (COND ((MUL-OPR ty x opr cc)
          (AND (FACTOR ty cc trm ccc)
               (TERM-TAIL (ty (opr u trm)) ccc r c)))))

(DEFPROP MUL-OPR
  ((MUL-OPR type tokens rep rest))
  DOC)

(PROCEDURE MUL-OPR ONERES)
(\- (MUL-OPR INTEGER (CONS * c) * c))
(\- (MUL-OPR INTEGER (CONS DIV c) \% c))
(\- (MUL-OPR INTEGER (CONS MOD c) REMAINDER c))
(\- (MUL-OPR BOOLEAN (CONS AND c) AND c))

(DEFPROP FACTOR
  ((FACTOR type tokens rep rest))
  DOC)

(PROCEDURE FACTOR ONERES)
(\- (FACTOR BOOLEAN (CONS TRUE c) T c))
(\- (FACTOR BOOLEAN (CONS FALSE c) NIL c))
(\- (FACTOR BOOLEAN (CONS ODD tx) (ODD e) c)
    \- (PARSE tx ("EXPRESSION INTEGER e") c))
(\- (FACTOR BOOLEAN (CONS NOT tx) (NOT f) c)
    \- (FACTOR BOOLEAN tx f c))
(\- (FACTOR ty (CONS /* (tx) e) c)
    \- (PARSE tx ("EXPRESSION ty e") c))

- 12-17 -
(DEFPROP VAR-IDENT
  ((VAR-IDENT ty var))
  DOC)

(PROCEDURE VAR-IDENT)

(DEFPROP PARSE
  ((PARSE tokens items rest)
   (An item may be a token or (syncat var) or (syncat parm var))
   DOC)
  .)

(PROCEDURE PARSE ONERES)

(DEFPROP REPEATO
  ((REPEATO cntrl tokens rep rest)
   (cntrl is (syncat sep) or (syncat parm sep))
   (Yields (<syncat><sep>)(<syncat>)))
  DOC)

(PROCEDURE REPEATO ONERES)
Note that procedures with more than one assertion are specified to be **ONERES**. That this is appropriate depends upon two circumstances. First, the grammar is unambiguous (we made it that way). Second, we expect that "tokens" really will be a list of atoms. For the auxiliary procedures **PARSE** and **REPEATO** the appropriateness of **ONERES** follows from the fact that the argument expressions in calls of these procedures are always specified in sufficient detail that only one assertion will apply.

**REPEATO**, which has the appearance of a parameterized category, handles constructs of the form "zero or more occurrences of 'syncat' (possibly with parameter) separated by 'sep'". The "representation" it produces is an expression having the form of a list of representations, and is used as the tail of some larger expression. The assertion dealing with compound statements illustrates the use of **REPEATO**.

The treatment of simple expressions (relation **SIMPLE-EXPR**) is complicated by the necessity of associating unparenthesized expressions to the left. "X + Y - Z", for example, means 

\[ (X + Y) - Z \]

To accomplish this we introduce the auxiliary relation **SIMPLE-TAIL**, whose parameter includes the representation of the previous portion of the expression being compiled. The object representation of "X + Y - Z" is

\[ \text{VAR being the run-time function which evaluates variables. TERM is handled similarly.} \]

we have implemented AND and OR using the corresponding LISP functions, which is not entirely proper, as LISP uses "short-cut" evaluation, unlike PASCAL. No real harm results, though, since our restricted language admits no expressions with side effects.

**12.2.3 Using The Compiler.**

One can use the compiler by simply invoking a query, as

\[ \#(\text{THE (r c) (STATEMENT * (X := Y + Z + 1 ;) r c)}) \]
Note that we can compile a single expression as easily as an entire program -- a useful feature for the language experimenter.

To make matters a little easier we have written some simple LISP programs to manage administrative chores, these being included with the run-time support programs. For our purposes a program is just a statement, followed perhaps by a terminating character such as ".", what follows is an example involving a less trivial program, the fast exponentiation algorithm.

*OPRINTIC FASTEXP*

(BEGIN

Y := 1, ;
Z := X ;
K := I ;
WHILE K > 0, DO
  IF UDD (K) THEN
    BEGIN Y := Y * Z ; K := K - 1. END
  ELSE
    BEGIN Z := Z * Z ; K := K DIV 2. END
  END .
NIL
*(COMPILE FASTEXP?)*

COMPILED
*(SPRINT OBJECT 1)*

(PROGN
 (:= Y 1.)
 (:= Z (VAR X))
 (:= K (VAR I))
 (PROG NIL
   LOOP: (COND
     ((> (VAR K) 0.)
       (COND ((UDD (VAR K))
         (PROGN
           (:= Y (* (VAR Y) (VAR Z)))
           (:= K (- (VAR K) 1.))))
         (PROGN
           (:= Z (* (VAR Z) (VAR Z)))
           (:= K (Z (VAR K) 2.*))))
       (GO LOOP:))))
NIL
*(DEEP X 2 1 13)*

Deposited
*(RUN OBJECT)*

- 12-20 -
The object program which results is left as the value of the identifier OBJECT, which we find to be a LISP version of the algorithm. The function DEP (for DEPosit) is used to preset the necessary variables, RUN executes the object program, and EXM is used afterwards to EXamine the outcome. Values of program variables are actually stored as PVAL properties of the variable identifiers, thus avoiding any possibility of collision with LISP identifier values, which are VALUE properties.

The techniques used in the run-time system are too primitive to serve for the implementation of more sophisticated languages, particularly those including procedures, but the compiler itself suffers no such defect. One point regarding the compiler bears further discussion. We have chosen, at the expense of some complication in the logic, to write the compiler so as to avoid backtracking. That this is the case is apparent from the use of ONERES. The result is a faster compiler than would be obtained with backtracking, but beyond this, when enlarging the language to encompass declarations we have the option of using imperative techniques for symbol table management. With backtracking, hence "concurrent" exploration of the deduction tree, this option would be lost.
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The file DOC.LSP (compiled version DOC.LAP) contains a collection of LISP programs which define a simple facility for documenting LISP systems on-line. The method used is to associate "documentation" with identifiers under the property DOC. This documentation may be any list structure whatever, although the package supports certain elementary conventions regarding function documentation. There are, in addition, a few functions which assist in the preparation of nicely formatted files with function definitions and documentation.

A.1 FUNCTIONS FOR DEFINING DOCUMENTATION

The package contains functions for defining documentation properties in general, as well as special functions for function documentation.

(DD "I" "DOC") [FEXPR]

Inserts documentation (DOC) on the property list of the identifier I. DOC may consist of several items, as in (DD FO0 (This is a) (silly example)), which results in ((This is a) (silly example)) as the documentation of FO0.

(DFO "F" "DOC") [FEXPR]

Inserts documentation for a previously defined function F. DFO automatically inserts the argument list and function type (EXPR, FEXPR or MACKU) at the front of the documentation so as to produce standard function documentation. As with DD, DOC may consist of several items. DFO returns the function name F. If F is not, in fact, a function, DFO acts like DD, but returns the list (DD F) to inform the user of its action.

The style of documentation produced by DFO is considered standard in the system, and three functions are provided for defining functions and documentation simultaneously.
(DEF "FN" "ARGS" ("DOC") "BODY")  (FEXPR)

Defines an EXPR named FN with argument list ARGS, documentation
DOC and body BODY, which is typed just as for DE. The
documentation must be a single item (usually a list) as signified
by the parentheses above, to which the argument list and type
(EXPR) will be added automatically. DE actually uses DE to
insert the definition, so newly defined functions are added to
the file SAVE. DOE returns FN, or (FN REDEFINED), if FN was
previously defined to be a function.

(DEF "FN" "ARGS" ("DOC") "BODY")  (FEXPR)

Is like DOE, except that the function thus defined is of type
EXPR.

(DEF "FN" "ARGS" ("DOC") "BODY")  (FEXPR)

Is like DOE, except that the function thus defined is of type
MACRO.

A.2 FUNCTIONS FOR PRINTING DOCUMENTATION

(DEF "I1" "I2" ...)  (MACRO)

Prints the documentation for I1, I2, ... (if any) in DEFPROP
format, using PP to obtain nice layout and indentation. DOC
returns an identifier with vacuous PNAME. Messages indicating
that some identifier "has no properties on PRETTYPROPS" simply
indicate the absence of a DOC property.

(PPDOC X)  (EXPR)

PPDOC X)

Controls the printing of documentation by PP. (PPDOC T) enables
printing of documentation, while (PPDOC NIL) disables printing of
documentation. PPDOC returns the new value of PRETTYPROPS, which
is the list of properties printed by PP. GRINDDOC is synonymous
with PPDOC.

A.3 FUNCTIONS FOR EDITING DOCUMENTATION

The editing functions are akin to EDITF in that editing commands
may be included in the function call or typed outside the call on
the same line, if one wishes.

(EDITU "I" "CONS")  (FEXPR)
Edits the documentation of I. COMS is an optional sequence of editing commands. EDITD returns I.

(EDITFD "FN" "COMS")  [FEXPR]

Edits the documentation and definition of the function FN simultaneously as a list having the form

(DOC documentation type definition)

which looks like a segment of the property list of FN. Although this list can be edited any way one likes, the type of the function (as specified in its property list) can not actually be changed, nor can the documentation property be removed. Upon exit from the editor the function definition and documentation are checked to insure that arguments and type agree and, if not, a message to that effect is printed and one is returned to the editor. If FN is not documented EDITFD prints $EDITF and runs EDITF. If FN is documented but not a function, it prints $EDITD and runs that function. EDITFD returns FN.

A.4 FUNCTIONS FOR GENERATING FILES

The functions described below provide means for writing files with linelengths specified at the terminal. This is particularly useful when generating files of LISP function definitions which are to be incorporated in papers typed on pages of normal size. These functions follow certain common conventions. In each case, the file generated by the function is written on device DSK; and the name of the file is the first argument. The name should be either an atom or a dotted pair. The linelength with which the file is written is the second argument and should be an integer, usually in the range 60 to 124.

(WRITEPROGS "FILE" "LENGTH" "FLNM")  [FEXPR]

GRINDEFS the identifiers which are MEMBERS of FLNM on FILE with linelength LENGTH. MEMBERS of FLNM which are not identifiers are simply printed. FLNM should be a file name defined for BUILD, as might be constructed with ADDTO. The properties recorded in the file are determined by the current value of GRINPROPS, as always. The resulting file can be read with DSKIN, but does not include the sort of file definition information written by BUILD. WRITEPROGS returns FILE.

(WRITEDOC "FILE" "LENGTH" "FLNM")  [FEXPR]

Generates a file much like WRITEPROGS, except that only DOC properties are recorded in the file.

- A-3 -
Generates a file with linelength LENGTH, the contents of the file being written by the evaluation of OPERATION. OPERATION may be any expression whatever whose evaluation results in printing. The local variables of WRITEANY all have the form #...#, so these are unlikely to interfere with global variables appearing in OPERATION. WRITEANY returns the value of OPERATION.

A.5 HINTS ON USING THE PACKAGE

Since SPRINT does not work particularly well on long lists of atoms, it is usually wise to divide narrative sections of the documentation into lists of manageable length. The next section gives some examples.

When developing a collection of LISP programs it seems most convenient to perform (GRINDOC T), so that BUILD will write documentation in the files it creates, which documentation will then be retrieved when the resulting files are read with DSKIN.

One may not wish to include all the documentation in a "production" system, since this could require a good deal of storage. In such circumstances one should BUILD after (GRINDOC NIL) and write a separate documentation file using WRITEDOC. Large linelengths are suggested for files not intended for publication. One can imagine other ways of using the package as well.

Files containing documentation may be compiled in the usual way, in which case the documentation will appear in the LAP file, where it can be read by DSKIN. Files written by WRITEPROGS or WRITEDOC are likely to be incorporated in RUNOFF source files. RUNOFF will produce the expected result if the text from the program file is preceded by the command

\*
inf.ts 3,10,24,32,40,48,56
*

One will usually need to insert skip commands in place of the blank lines appearing in files written by these programs. Beware of characters such as ']' which are of special significance to RUNOFF, and also the \* which PRINT generates when atoms cross the end of a line.

- A-4 -
A.6 AN EXAMPLE

We give now an example consisting of a few of the functions in the file DOC.LSP, along with the documentation which is associated with those functions (and included in DOC.LSP). This listing was generated by performing

(*(PPDOC T)
 (DOC SPECIAL (READMACRO . PP-RMACS) EXPR FEXPR MACRO (VALUE . PP-VAULE) PRINTMACRO)
 *(ADDTO EXAMPLE OD DOC WRITEPROGS)
 (NEW FILE EXAMPLE)
 NIL
 *(WRITEPROGS (EXAMPLE . TXT) 60. EXAMPLE)
 (EXAMPLE . TXT)
 *)

The resulting file is:

*(FSUBR (DO WRITEPROGS))
 *(DEFPROP DOC
 (LAMBDA (L)
 MACRO
 (DOC "II" "12" ---)
 (Prints documentation of II 12 --- in DEFPROP format if defined))
 DOC)
 *(DEFPROP DOC
 (LAMBDA (L) (SUBST (CDR L) 'X '((PP (P: (DOC) . X))))
 MACRO)
 *(DEFPROP DO
 (LAMBDA (L)
 FEXPR
 (DO "I" "DOC")
 (Define documentation for I))
 DOC)
(DEFPROP DOC
  (LAMBDA (L) (PUTPROP (CAR L) (CDK L) 'DOC) (CAR L))
  FEXPR)

(DEFPROP WRITEPROGS
  (LAMBDA (F)
    FEXPR
    (WRITEPROGS "FILE" "LENGTH" "FLNM")
    (GRINDEFs all atoms which are MEMBERS of FLNM on DSK:FILE)
    (PRINTs non-atomic MEMBERS of FLNM)
    (Line-length given by LENGTH)
    (FLNM should be a filename defined for BUILD))
  DOC)

(DEFPROP WRITEPROGS
  (LAMBDA (F)
    (PRUG (DLDC LWB L)
      (EVAL (LIST (FUNCTION OUTPUT)
        'WRITEPROG
        'OSK:
        (CAR F)))
      (SETQ DLDC (OUTC 'WRITEPROG NIL))
      (SETQ LWB (LINELENGTH NIL))
      (LINELENGTH (CADR F))
      (SETQ L (GET (CADDR F) 'MEMBERS))
      LOOP (COND
        [L (COND (((ATOM (CAR L))
                 (EVAL (LIST 'GRINDEF (CAR L))))
                 ([TERPRI (PRINT (TERPRI (CAR L))))])
          (SETQ L (CDR L))
          (GO LOOP))]
      (LINELENGTH LWB)
      (OUTC DLDC T)
      (RETURN (CAR F))))
    FEXPR)