```
/*evans*/
/*Evan's Geometric Analogy Program - Rational Reconstruction*/
/*Alan Bundy 26.10.79*/


/*top level program*/
evans(FigA,FigB,FigC,AnsList,Ans) :-
        findrule(FigA,FigB,Rule), rule_is(Rule),
        applyrule(Rule,FigC,AnsObjs,AnsRels,Sims),
        ans_desc(AnsObjs,AnsRels,Sims),
        selectresult(FigC,AnsList,AnsObjs,AnsRels,Sims,Ans),
        ans_is(Ans).


/*find rule given figures*/
findrule(FigA,FigB,Rule) :-
        relations(FigA,Source), relations(FigB,Target),
        objects(FigA,Alist), objects(FigB,Blist),
        similarities(FigA,FigB,Triples),
        select_set(Triples,Matches),
        takeaway1(Alist,Matches,Removals),
        takeaway2(Blist,Matches,Adds),
        makerule(Removals,Adds,Matches,Source,Target,Rule).

/*applyrule to figc to produce answer*/
applyrule(rule(Removals,Adds,Matches,Source,Target),
                FigC, AnsObjs,Target,Matches) :-
        relations(FigC,FigDesc), objects(FigC,ObList),
        seteq(FigDesc,Source),
        maplist(second,Matches,NewList), append(NewList,Adds,AnsObjs).

/* Select Result from those provided */
selectresult(FigC,[FigN|Rest],AnsObjs,AnsRels,AnsSims,FigN) :-
        relations(FigN,NRels), seteq(NRels,AnsRels),
        similarities(FigC,FigN,NSims), seteq(NSims,AnsSims),
        objects(FigN,NObjs), seteq(NObjs,AnsObjs).

selectresult(FigC,[FigN|Rest],AnsObjs,AnsRels,AnsSims,Ans) :-
        selectresult(FigC,Rest,AnsObjs,AnsRels,AnsSims,Ans).


/*select legal subset of similarity triples for matches*/
select_set(Triple,Match) :- select_set1([],[],Triple,Match).

select_set1(Aused,Bused,[],[]).

select_set1(Aused,Bused,[[Aobj,Bobj,Trans]|Rest],[[Aobj,Bobj,Trans]|Rest1]) :-
        not(member(Aobj,Aused)), not(member(Bobj,Bused)),
        select_set1([Aobj|Aused],[Bobj|Bused],Rest,Rest1).

select_set1(Aused,Bused,[[Aobj,Bobj,Trans]|Rest],Rest1) :-
        select_set1(Aused,Bused,Rest,Rest1).


/*take away the triples from the list*/
takeaway1(List,Triples,Ans) :-
        maplist(first,Triples,Firsts), subtract(List,Firsts,Ans).

takeaway2(List,Triples,Ans) :-
        maplist(second,Triples,Seconds), subtract(List,Seconds,Ans).
```

```
/* First and second elements of a list */
first([A,B,C],A).
second([A,B,C],B).


/*make rule from descriptions inherited from figs a & b*/
makerule(Removals,Adds,Matches,Source,Target,Rule) :-
        maplist(first,Matches,Spairs), maplist(second,Matches,Tpairs),
        append(Removals,Spairs,L1), append(L1,Tpairs,L2),
        append(L2,Adds,Consts),
        unbind(Consts,Substs),
        subst(Substs,rule(Removals,Adds,Matches,Source,Target),Rule).

/*find corresponding variable for each constant and produce substitution*/
unbind([],true).

unbind([Const|Rest],Const=X & Rest1) :-
        unbind(Rest,Rest1).


/* Messages */
/*----------*/

rule_is(rule(Removals,Adds,Matches,Source,Target)) :-
        writef('Rule is:
        remove: %t
        add: %t
        match: %t
        source: %t
        target: %t \n\n', [Removals,Adds,Matches,Source,Target]).

ans_desc(Objs,Rels,Sims) :-
        writef('Answer description is:
        objects: %t
        relations: %t
        similarities: %t \n\n', [Objs,Rels,Sims]).

ans_is(Ans) :-
        writef('Answer is %t\n\n\n',[Ans]).
```

```
/*figures*/
/*test descriptions for evans program*/
/*Alan Bundy 26.10.79*/

problem1(Ans) :- evans(figa,figb,figc,[fig1,fig2,fig3,fig4,fig5],Ans).
problem2(Ans) :- evans(figa,figb,figc,[fig1,fig2,fig3,fig4a,fig5],Ans).
problem3(Ans) :- evans(figa,figb,figc,[fig1,fig2,fig3,fig5],Ans).


objects(figa,[tri1,tri2]).
relations(figa,[[inside,tri1,tri2]]).

objects(figb,[tri3]).
relations(figb,[]).

similarities(figa,figb,[[tri2,tri3,direct], [tri1,tri3,[scale,2]]]).

objects(figc,[square,circle]).
relations(figc,[[inside,square,circle]]).


objects(fig1,[circle2,circle3]).
relations(fig1,[[inside,circle2,circle3]]).
similarities(figc,fig1,[[circle,circle3,direct],
                        [circle,circle2,[scale,half]]]).

objects(fig2,[square2]).
relations(fig2,[]).
similarities(figc,fig2,[[square,square2,direct]]).

objects(fig3,[tri4,circle4]).
relations(fig3,[[inside,tri4,circle4]]).
similarities(figc,fig3,[[circle,circle4,direct]]).

objects(fig4,[circle5]).
relations(fig4,[]).
similarities(figc,fig4,[[circle,circle5,direct]]).

    ects(fig4a,[square3]).
relations(fig4a,[]).
similarities(figc,fig4a,[[square,square3,[scale,2]]]).

objects(fig5,[tri5]).
relations(fig5,[]).
similarities(figc,fig5,[]).
```

```
yes
| ?- problem1(A).
Rule is:
        remove: [_527]
        add: []
        match: [[_537,_547,direct]]
        source: [[inside,_527,_537]]
        target: []

Answer description is:
        objects: [_547]
        relations: []
        similarities: [[circle,_547,direct]]

Answer is fig4



A = fig4

yes
| ?- problem2(A).
Rule is:
        remove: [_527]
        add: []
        match: [[_537,_547,direct]]
        source: [[inside,_527,_537]]
        target: []

Answer description is:
        objects: [_547]
        relations: []
        similarities: [[circle,_547,direct]]

Rule is:
        remove: [_527]
        add: []
        match: [[_537,_547,[scale,2]]]
        source: [[inside,_537,_527]]
        target: []

Answer description is:
        objects: [_547]
        relations: []
        similarities: [[square,_547,[scale,2]]]

Answer is fig4a



A = fig4a

yes
| ?- problem3(A).
Rule is:
        remove: [_527]
        add: []
        match: [[_537,_547,direct]]
        source: [[inside,_527,_537]]
```

```
        target: []

Answer description is:
        objects: [_547]
        relations: []
        similarities: [[circle,_547,direct]]

Rule is:
        remove: [_527]
        add: []
        match: [[_537,_547,[scale,2]]]
        source: [[inside,_537,_527]]
        target: []

Answer description is:
        objects: [_547]
        relations: []
        similarities: [[square,_547,[scale,2]]]

Rule is:
        remove: [_381,_391]
        add: [_401]
        match: []
        source: [[inside,_381,_391]]
        target: []

Answer description is:
        objects: [_401]
        relations: []
        similarities: []

Answer is fig5


A = fig5

yes
| ?- core      49664  (20480 lo-seg + 29184 hi-seg)
    p     15360 =  14048 in use +   1312 free
g obal    1187 =     16 in use +   1171 free
local     1024 =     16 in use +   1008 free
trail      511 =      0 in use +    511 free
    0.06 sec. for 1 GCs gaining 144 words
    0.08 sec. for 4 local shifts and 12 trail shifts
    1.77 sec. runtime
```

DEPARTMENT OF ARTIFICIAL INTELLIGENCE
PROLOG PROGRAM LIBRARY
PROGRAM SPECIFICATION

Program Name	The Evans Geometric Analogy Program
Source	Alan Bundy
Date of Issue	8 May 1981

1. Description

This is a rational reconstruction of the second part of Evan's program to solve Geometric Analogy Problems [Evans 64, Bundy 80]. It uses the descriptions of geometric figures to form a rule; applies this rule to form the description of [illegible] answer figure and then uses this description to select an answer from those provided. The first part of Evan's program, in which figure descriptions are formed from a cartesian representation, is not attempted.

2. Method of Use

To use the program, type

```
run PLL: UTIL
```

and in response to the prompt type

```
consult('PLL: EVANS').
```

The top level predicate, **evans**, will then be available. **evans** takes 5 arguments: the names of figures A, B, C; a list of possible answer figures and a variable. If it succeeds then **evans** binds this variable to one of the answer figures. As it runs **evans** prints descriptions of the rules it forms and the answer figures these suggest.

For the program to work, assertions must be present in the database describing the figures A, B, C and the possible answer figures and relating objects in figure A to objects in figure B and objects in figure C to objects in each of the answer figures. Some examples of the sort of assertions required are given in file PLL: FIGURE and a selection are reproduced in example 2-1 below.

```
          objects(figa,[tri1,tri2]).
          relations(figa,[[inside,tri1,tri2]]).

          objects(figb,[tri3]).
          relations(figb,[]).

          similarities(figa,figb,[[tri2,tri3,direct],
                                  [tri1,tri3,[scale,2]]]).

          objects(figc,[square,circle]).
          relations(figc,[[inside,square,circle]]).
```

Figure 2-1: Assertions to Describe Input Figures

- **objects** takes a figure name and a list of the objects occuring in the figure.
- **relations** takes a figure name and a list of symbolic descriptions describing the figure.
- **similarities** takes two figure names and a list of similarities betwen objects in the two figures.

The file, PLL: FIGURE, can be input by typing

```
     consult('PLL: FIGURE').
```

The particular assertions defining figa, figb and figc given above, plus similar ones for fig1-5 will then be in the database and the call

```
     evans(figa,figb,figc,[fig1,fig2,fig3,fig4,fig5        ( )

,Ans).]  will then succeed and bind Ans to fig4.
```

3. How it Works

The top level predicate,evans, is defined as follows (omitting print messages):

```
     evans(FigA,FigB,FigC,AnsList,Ans) :-
               find_rule(FigA,FigB,Rule),
               apply_rule(Rule,FigC,AnsObjs,AnsRels,Sims),
               select_result(FigC,AnsList,AnsObj,AnsRels,Sims,Ans).
```

find_rule takes the names of figures A and B and forms a description of the rule relating them. **apply_rule** takes this rule and applies it to figure C, producing a description of the answer figure and similarities between figure C and the answer figure. **select_result** takes these descriptions and compares them to the available answer figures until it finds a match.

find_rule is defined as follows:

```
find_rule(FigA,FigB,Rule) :-
        relations(FigA,Source), relations(FigB,Target),
        objects(FigA,Alist), objects(FigB,Blist),
        similarities(FigA,FigB,Triples),
        select_set(Triples,Matches),
        takeaway1(Alist,Matches,Removals),
        takeaway2(Blist,Matches,Adds),
        make_rule(Removals,Adds,Matches,Source,Target,Rule).
```

he first five subprocedures pick up the prestored descriptions of the two figures. The predicate **select_set** picks a legal subset, Matches, of the similarity descriptions between objects in figure A and B. The rule description is based on this subset. On backup a different subset will be chosen and a different rule formed. Objects occuring in figure A, but not involved in a match are then selected by **takeaway1** and these become the Removals.

Similarly, **takeaway2** finds those objects occurring in figure B, but not involved in a match and these become the Adds. A rule description is then formed from these components by **make_rule**.

select_set and the two **takeaways** are fairly straightforward list manipulation, but **make_rule** requires some explanation. It is defined as follows:

```
make_rule(Removals,Adds,Matches,Source,Target,Rule) :-
        maplist(first,Matches,Spairs), maplist(second,Matches,Tpairs),
        append(Removals,Spairs,L1), append(L1,Tpairs,L2),
        append(L2,Adds,Consts),
        unbind(Consts,Substs),
        subst(Substs,rule(Removals,Adds,Matches,Source,Target),Rule).
```

A rule description consists of the predicate rule with five arguments:

- A list of objects to be removed;
- A list of objects to be added;
- A list of matches between objects;
- The relations in the source figure and

- The relations in the target figure.

These are pretty much as supplied to **make_rule** except that the actual constants inherited from the initial figure descriptions have to be changed for Prolog variables, so that the rule can be applied to figure C.

The first five subprocedures of the **make_rule** definition consist of making a list, Consts, of all the object names appearing in the Removal, Adds and Matches. A substitution is then built by **unbind**, in which each of these object names is associated with a different Prolog variable. **subst** then substitutes these variables for the objects in the rule description. The definitions of **unbind**, **first** and **second** are straightforward and the definitions of **maplist** and **subst** are given in the Mecho utility program library.

The definition of **apply_rule** is given below.

```
    apply_rule(rule(Removals,Adds,Matches,Source,Target),
                     FigC, AnsObjs,Target,Matches) :-
            relations(FigC,FigDesc), objects(FigC,ObList),
            seteq(FigDesc,Source),
            maplist(second,Matches,NewList), append(NewList,Adds,AnsObjs).
```

The first two subprocedures recover the description of figure C. The predicate **seteq** then pattern matches the relations in figure C with the rule source description, which binds the first half of the similarity descriptions in Matches. **seteq** is set equality and is described in the Mecho utilities library. The objects in the answer figure are calculated by appending the Adds to the objects in the second half of the similarity descriptions in Matches.

The definiton oif **select_result** is:

```
    select_result(FigC,[FigN|Rest],AnsObjs,AnsRels,AnsSims,FigN) :-
            relations(FigN,NRels), seteq(NRels,AnsRels),
            similarities(FigC,FigN,NSims), seteq(NSims,AnsSims),
            objects(FigN,NObjs), seteq(NObjs,AnsObjs).

    select_result(FigC,[FigN|Rest],AnsObjs,AnsRels,AnsSims,Ans) :-
            select_result(FigC,Rest,AnsObjs,AnsRels,AnsSims,Ans).
```

select_result is defined by recursion on the list of available answers. If it recurses to the ends of this list then it fails, and **evans** backs up to form a new rule. The second clause is merely the recursive step: the first clause makes the interesting comparisons. FigN is the first answer figure name in the list. In turn the relations, objects and similarities of FigN are found in the database, and these are compared with FigC using **seteq**. FigN is returned as the answer iff the clause succeeds. Otherwise **select_result** recurses.

4. Program Requirements

The Prolog system takes 30K words and PLL: UTIL, PLL: EVANS, PLL: FIGURE and working space require an additional 21K words. The following predicates are used by the program:

```
ans_desc(3)       ans_is(1)         append(3)         apply_rule(5)
evans(5)          find_rule(3)      first(2)          make_rule(6)
maplist(3)        member(2)         objects(2)        problem1(1)
problem2(1)       problem3(1)       relations(2)      rule_is(1)
second(2)         select_result(6)select_set(2)     select_set1(4)
seteq(2)          similarities(3) subst(3)          subtract(3)
takeaway1(3)      takeaway2(3)      unbind(2)         writef(2)
```

REFERENCES

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Bundy, A.
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Occasional Paper 22, Dept. of Artificial Intelligence,
Edinburgh, September, 1980.

[Evans 64]

Evans, T.G.
A heuristic program to solve geometric analogy problems.
J.S.C.C. , April, 1964.

```
/*winst1
Rational Reconstruction of Winston Learning Program
Alan Bundy 1.12.80
Version for functions */

/* Top Level Program - learn new concept */
/* ---------------------------------------- */

/*First time only accept an example */
winston(Concept) :- !,
        writef('Please give me an example of a %t \n', [Concept]),
        read(Ex), nl,
        make_rec(Concept,Ex,EObjs,ERec),
        maplist(gensym1(plato),EObjs,CObjs),
        make_subst(EObjs,CObjs,Subst),
        subst(Subst,ERec,CRec),
        maplist(add_ups,CRec,CDefn),
        assert(definition(Concept,CObjs,CDefn)),
        winston1(Concept).

/* Is grey area in definition eliminated? */
 inston1(Concept) :-
        definition(Concept,CObjs,CDefn),
        checklist(same,CDefn), !,
        writef('I have learnt the concept of %t now. \n', [Concept]).

/*Subsequently accept either examples or near misses */
winston1(Concept) :- !,
        writef('Please give me an example or near miss of a %t. \n', [Concept]),
        read(Ex), nl,
        writef('Is this example (yes./no.)? \n',[]),
        read(YesNo), nl,
        learn(Concept,Ex,YesNo),
        winston1(Concept).

/* add default upper bounds in concept record */
add_ups(record(Args,Name,Posn), define(Args,Name,[],Posn)).


/* slight modify to gensym, so it can be used in maplist */
 ensym1(Prefix,_,NewConst) :- !, gensym(Prefix,NewConst).


/* are upper and lower bound of concept definition the same? */
same(define(Args,Name,Posn,Posn)).

/* learn from this example or near miss */
learn(Concept, Example, YesNo) :- !,
        definition(Concept,CObjs,CDefn),
        make_rec(Concept,Example,EObjs,ERec),
        classify(CObjs,EObjs,CDefn,ERec,Diff,Verdict),
        learn1(Concept,Diff,YesNo,Verdict).


/* Make  records from list of relations */
/* ---------------------------------------- */

make_rec(Concept,Example,EObjs,ERec) :- !,
        example(Example,Relns),
        maplist(consts_in,Relns,CL), flatten(CL,EObjs),
```

```
        maplist(convert,Relns,ERec).

/* Find all constants in terms */
consts_in([],[]).

consts_in(N,[]) :-
        integer(N), !.

consts_in(Const,[Const]) :-
        atom(Const), !.

consts_in(Exp,Consts) :-
        Exp =.. [Sym|Args], maplist(consts_in,Args,CL),
        flatten(CL,Consts).

/*Flatten List */
flatten([],[]).

flatten([Hd|Tl],Ans):-
        flatten(Tl,Rest), union(Hd,Rest,Ans).


/* Convert input relation style into internal representation as predicate tree */
convert(Reln, record(Args,Name,ExPosn)) :-
        Reln =.. [Pred|Args],
        length(Args,N),
        tree(Name,N,Tree),
        position(Pred,Tree,ExPosn).

/* Find Position of Node in Tree */
position(Node,Tree,[]) :-
        Tree =.. [Node|SubTrees].

position(Node,Tree,[N|Posn]) :-
        Tree =.. [Root|SubTrees],
        nth_el(N,SubTrees,SubTree),
        position(Node,SubTree,Posn).

/* find nth element of list */
nth_el(1,[Hd|Tl],Hd).

nth_el(N,[Hd|Tl],El) :-
        nth_el(PN,Tl,El), N is PN + 1.

/* Is this example, non-example or in grey area, by my definition? */
/* --------------------------------------------------------------- */

classify(CObjs,EObjs,CDefn,ERec,BestDiff,Verdict) :- !,
        findall(Diff, make_diff(CObjs,EObjs,CDefn,ERec,Diff), Diffs),
        best(Diffs,BestDiff),
        verdict(BestDiff,Verdict).

/* Find the difference between example and concept */
make_diff(CObjs,EObjs,CDefn,ERec,Diff) :- !,
        perm(EObjs,EObjs1), make_subst(EObjs1,CObjs,Subst),
        subst(Subst,ERec,ERec1),
        pair_off(CDefn,ERec1,Diff).

/*Pair off concept definition and example record to make differences */
pair_off([],[],[]) :- !.
```

```
pair_off([],ERec,Diff) :- !,
        maplist(new_defn,ERec,Diff).

pair_off(CDefn,[],Diff) :- !,
        maplist(extra_rec,CDefn,Diff).

pair_off([define(Args,Name,UpPosn,LowPosn) | CDefn],
         ERec,
         [differ(Args,Name,UpPosn,ExPosn,LowPosn,Verdict) | Diff]) :-
                select(record(Args,Name,ExPosn),ERec,Rest), !,
                compare(UpPosn,ExPosn,LowPosn,Verdict),
                pair_off(CDefn,Rest,Diff).

/* invent new bits of definition as necessary */
new_defn(record(Args,Name,ExPosn),
         differ(Args,Name,[],ExPosn,DfPosn,Verdict)) :-
                default_posn(Name,DfPosn),
                compare([],ExPosn,DfPosn,Verdict).

/* invent extra bits of example record as necessary */
 xtra_rec(define(Args,Name,UpPosn,LowPosn),
          differ(Args,Name,UpPosn,DfPosn,LowPosn,Verdict)) :-
                default_posn(Name,DfPosn),
                compare(UpPosn,DfPosn,LowPosn,Verdict).


/* Find position of default predicate on tree */
default_posn(TreeName,Posn) :-
        default(TreeName,Pred), !,
        tree(TreeName,_,Tree), position(Pred,Tree,Posn).

default_posn(TreeName,[]).


/* Compare positions in tree to give verdict */
compare(U,E,L,yes) :- append(L,_,E), !.
compare(U,E,L,grey) :- append(U,_,E), !.
compare(U,E,L,no) :- !.

 * Find best difference and return it */
est(Diffs,Diff) :- !,
        maplist(score,Diffs,Scores),
        lowest(Diffs,Scores,Diff,Score).

/* Return difference with lowest score */
lowest([Diff],[Score],Diff,Score) :- !.

lowest([Diff1|Diffs], [Score1|Scores], Diff2, Score2) :-
        lowest(Diffs,Scores,Diff2,Score2), Score2<Score1, !.

lowest([Diff|Diffs], [Score|Scores], Diff, Score) :- !.

/* Find score of difference */
score(Diff,Score) :- !,
        maplist(score1,Diff,Scores),
        sumlist(Scores,Score).

/* Find score of individual differ */
score1(differ(_,_,_,_,_,yes), 0) :- !.
```

```
score1(differ(_,_,_,_,_,grey), 1) :- !.
score1(differ(_,_,_,_,_,no), 2) :- !.

/* add up all the numbers in a list */
sumlist([], 0).
sumlist([N|Rest], Total) :- !,
        sumlist(Rest,SubT), Total is SubT + N.

/* Make a substitution for replacing all members of one list
by corresponding members of another list */
make_subst([],[],true).

make_subst([X|XRest],[Y|YRest],X=Y & Subst) :-
        make_subst(XRest,YRest,Subst).


/* Decide whether example falls inside definition on basis of differs */
verdict(Diff,yes) :- checklist(verdict1(yes),Diff), !.
verdict(Diff,no) :- some(verdict1(no),Diff), !.
verdict(Diff,grey) :- some(verdict1(grey),Diff), !.

 * verdict on individual differ */
verdict1(V, differ(_,_,_,_,V)).


/* adjust definition  appropriately */
/* -------------------------------- */

/* if new example found */
learn1(Concept,Diff,yes,grey) :- !,
        writef('This is a new sort of %t. \n', [Concept]),
        maplist(lub,Diff,New),
        retract(definition(Concept,CObjs,Old)),
        assert(definition(Concept,CObjs,New)).

/* if near miss found */
learn1(Concept,Diff,no,grey) :- !,
        writef('This limits my idea of %t. \n', [Concept]),
        one_of(exclude,Diff,Diff1),
        maplist(diff_to_defn,Diff1,New),
        retract(definition(Concept,CObjs,Old)),
        assert(definition(Concept,CObjs,New)).

/* if nothing new is discovered */
learn1(Concept,Diff,Agree,Agree) :- !,
        writef('I have seen one of these before. \n',[]).

/* or if contradiction is discovered */
learn1(Concept,Diff,Agree,Disagree) :- !,
        writef('Uh Oh, somethings gone wrong. I will think again.\n',[]),
        fail.

/* Move lower definition up a bit to include new example */
lub(differ(Args,Name,UpPosn,ExPosn,Old,grey),
    define(Args,Name,UpPosn,New)) :- !,
        common(ExPosn,Old,New).

/* Lower definition already includes new example */
lub(differ(Args,Name,UpPosn,ExPosn,LowPosn,yes),
    define(Args,Name,UpPosn,LowPosn)) :- !.
```

```
/* Move upper definition down a bit to exclude near miss */
exclude(differ(Args,Name,Old,ExPosn,LowPosn,grey),
    differ(Args,Name,New,ExPosn,LowPosn,grey)) :- !,
        common(ExPosn,LowPosn,Comm), append(Comm,[N|_],LowPosn),
        append(Comm,[N],New).

/* Take unnecessary bits out of difference  */
diff_to_defn(differ(Args,Name,UpPosn,ExPosn,LowPosn,Verdict),
             define(Args,Name,UpPosn,LowPosn)).


/* Find common initial sublist of two lists */
common([N|Rest1], [N|Rest2], [N|Rest]) :- !,
        common(Rest1,Rest2,Rest).

common(List1,List2,[]) :- !.

/* change just one member of list */
one_of(Prop, [Old|Tl], [New|Tl]) :- apply(Prop,[Old,New]).
one_of(Prop, [Hd|Old], [Hd|New]) :- one_of(Prop,Old,New).


/* Find out what grey areas still exist in concept */
grey(Concept) :- !,
        writef('Grey areas in %t are:\n',[Concept]),
        definition(Concept,CObjs,CDefn),
        checklist(grey1,CDefn).

grey1(define(Args,Name,Posn,Posn)) :- !.

grey1(Defn) :- !,
        write(Defn), nl.
```

```
/*arch
Winston arch domain
Alan Bundy 5.12.80
use with winston */

/* space of description trees */
space(arch,[shapetree,touchtree,orienttree,directiontree,supporttree]).


/* description tree */

tree(shapetree,1,shape(prism(wedge,block),pyramid)).

tree(touchtree,2,touchrel(separate,touch(marries,abuts))).
default(touchtree,separate). % default predicate

tree(orienttree,1,orientation(lying,standing)).

tree(directiontree,2,direction(leftof,rightof)).

tree(supporttree,2,undef(supports,unsupports)).


/* Examples and near misses */

example(arch1, [block(a), block(b), block(c),
        standing(a), standing(b), lying(c),
        leftof(a,b),
        supports(a,c), supports(b,c),
        marries(a,c), marries(c,a), marries(b,c), marries(c,b)]).

example(arch2, [block(a), block(b), wedge(c),
        standing(a), standing(b), lying(c),
        leftof(a,b),
        supports(a,c), supports(b,c),
        marries(a,c), marries(c,a), marries(b,c), marries(c,b)]).

example(arch3, [block(a), block(b), block(c),
        standing(a), standing(b), lying(c),
        leftof(a,b),
        supports(a,c), supports(b,c),
        abuts(a,c), abuts(c,a), abuts(b,c), abuts(c,b)]).

example(archn1, [block(a), block(b), block(c),
        standing(a), standing(b), lying(c),
        leftof(a,b),
        supports(a,c), supports(b,c),
        marries(a,c), marries(c,a), marries(b,c), marries(c,b),
        marries(a,b), marries(b,a)]).

example(archn2, [block(a), block(b), block(c),
        standing(a), standing(b), lying(c),
        leftof(a,b),
        marries(a,c), marries(c,a), marries(b,c), marries(c,b)]).

example(archn3, [block(a), block(b), block(c),
        standing(a), standing(b), lying(c),
        leftof(a,b)]).
```

Sample of Session with Prolog Winston

```
| ?- winston(arch).
Please give me an example of a arch
|: arch1.

Please give me an example or near miss of a arch.
|: arch2.

Is this example (yes./no.)?
|: yes.

This is a new sort of arch.
Please give me an example or near miss of a arch.
|: arch3.

Is this example (yes./no.)?
|: yes.

This is a new sort of arch.
Please give me an example or near miss of a arch.
|: archn1.

Is this example (yes./no.)?
|: no.

This limits my idea of arch.
Please give me an example or near miss of a arch.
|: archn2.

Is this example (yes./no.)?
|: no.

This limits my idea of arch.
Please give me an example or near miss of a arch.
|: archn3.

Is this example (yes./no.)?
|: no.

I have seen one of these before.
Please give me an example or near miss of a arch.



| ?- grey(arch).
Grey areas in arch are:
define([plato1],shapetree,[],[1,2])
define([plato2],shapetree,[],[1])
define([plato3],shapetree,[],[1,2])
define([plato1,plato2],touchtree,[],[2])
define([plato2,plato1],touchtree,[],[2])
define([plato3,plato1],touchtree,[],[1])
define([plato2,plato3],touchtree,[],[2])
define([plato3,plato2],touchtree,[],[2])
define([plato1],orienttree,[],[2])
define([plato2],orienttree,[],[1])
define([plato3],orienttree,[],[2])
define([plato1,plato3],directiontree,[],[1])
```

```
define([plato3,plato2],supporttree,[],[1])
```

```
/*arch1
Winston arch domain
Alan Bundy 5.12.80
use with winston
version with functions */


/* description trees */

tree(shapetree,1,shape(prism(wedge,block),pyramid)).

tree(touchtree,2,touchrel(separate,touch(marries,abuts))).
default(touchtree,separate). % default predicate

tree(orienttree,1,orientation(lying,standing)).

tree(directiontree,2,direction(leftof,rightof)).

tree(supporttree,2,undef(supports,unsupports)).


/* Examples and near misses */

example(arch1, [block(lp(a)), block(rp(a)), block(tm(a)),
        standing(lp(a)), standing(rp(a)), lying(tm(a)),
        leftof(lp(a),rp(a)),
        supports(lp(a),tm(a)), supports(rp(a),tm(a)),
        marries(lp(a),tm(a)),  marries(rp(a),tm(a)) ]  ).

example(arch2, [block(lp(a)), block(rp(a)), wedge(tm(a)),
        standing(lp(a)), standing(rp(a)), lying(tm(a)),
        leftof(lp(a),rp(a)),
        supports(lp(a),tm(a)), supports(rp(a),tm(a)),
        marries(lp(a),tm(a)), marries(rp(a),tm(a)) ] ).

example(arch3, [block(lp(a)), block(rp(a)), block(tm(a)),
        standing(lp(a)), standing(rp(a)), lying(tm(a)),
        leftof(lp(a),rp(a)),
        supports(lp(a),tm(a)), supports(rp(a),tm(a)),
        abuts(lp(a),tm(a)), abuts(rp(a),tm(a)) ] ).

example(archn1, [block(lp(a)), block(rp(a)), block(tm(a)),
        standing(lp(a)), standing(rp(a)), lying(tm(a)),
        leftof(lp(a),rp(a)),
        supports(lp(a),tm(a)), supports(rp(a),tm(a)),
        marries(lp(a),tm(a)), marries(rp(a),tm(a)), marries(lp(a),rp(a)) ] ).

example(archn2, [block(lp(a)), block(rp(a)), block(tm(a)),
        standing(lp(a)), standing(rp(a)), lying(tm(a)),
        leftof(lp(a),rp(a)),
        marries(lp(a),tm(a)), marries(rp(a),tm(a)) ] ).

example(archn3, [block(lp(a)), block(rp(a)), block(tm(a)),
        standing(lp(a)), standing(rp(a)), lying(tm(a)),
        leftof(lp(a),rp(a))]).
```

HY o....

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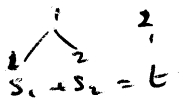
```
/* isol
Definition of Isolation space and examples for Winston Program
Alan Bundy 18.2.81 */

/* Predicate Trees */

tree(occurtree,2,occur_rel(freeof,contains(singleocc,multocc))).
default(occurtree,freeof).

tree(simtree,2,sim_rel(different(unrelated,inverse),ident)).
default(simtree,unrelated).
```



```
/* Examples and near Misses */

example(isol1, [singleocc(x,expr_at([1,2],before)),
                 freeof(x,expr_at([1,1],before)),
                 freeof(x,expr_at([2],before)),
                 ident(expr_at([1,1],before),expr_at([2,1],after)),
                 ident(expr_at([1,2],before),expr_at([1],after)),
                 ident(expr_at([2],before),expr_at([2,2],after)),
                 inverse(sym_at([1],before),sym([2],after)) ] ).

example(isol2, [singleocc(x,expr_at([1,2],before)),
                 contains(x,expr_at([1,1],before)),
                 freeof(x,expr_at([2],before)),
                 ident(expr_at([1,1],before),expr_at([2,1],after)),
                 ident(expr_at([1,2],before),expr_at([1],after)),
                 ident(expr_at([2],before),expr_at([2,2],after)),
                 inverse(sym_at([1],before),sym([2],after)) ] ).


example(isol3, [singleocc(x,expr_at([1,2],before)),
                 freeof(x,expr_at([1,1],before)),
                 contains(x,expr_at([2],before)),
                 ident(expr_at([1,1],before),expr_at([2,1],after)),
                 ident(expr_at([1,2],before),expr_at([1],after)),
                 ident(expr_at([2],before),expr_at([2,2],after)),
                 inverse(sym_at([1],before),sym([2],after)) ] ).

example(isol4, [singleocc(x,expr_at([1,2],before)),
                 freeof(x,expr_at([1,1],before)),
                 freeof(x,expr_at([2],before)),
                 different(expr_at([1,1],before),expr_at([2,1],after)),
                 ident(expr_at([1,2],before),expr_at([1],after)),
                 ident(expr_at([2],before),expr_at([2,2],after)),
                 inverse(sym_at([1],before),sym([2],after)) ] ).

example(isol5, [singleocc(x,expr_at([1,2],before)),
                 freeof(x,expr_at([1,1],before)),
                 freeof(x,expr_at([2],before)),
                 ident(expr_at([1,1],before),expr_at([2,1],after)),
                 different(expr_at([1,2],before),expr_at([1],after)),
                 ident(expr_at([2],before),expr_at([2,2],after)),
                 inverse(sym_at([1],before),sym([2],after)) ] ).


example(isol6, [singleocc(x,expr_at([1,2],before)),
                 freeof(x,expr_at([1,1],before)),
                 freeof(x,expr_at([2],before)),
```

```
                ident(expr_at([1,1],before),expr_at([2,1],after)),
                ident(expr_at([1,2],before),expr_at([1],after)),
                different(expr_at([2],before),expr_at([2,2],after)),
                inverse(sym_at([1],before),sym([2],after)) ] ).

example(isol7, [singleocc(x,expr_at([1,2],before)),
                freeof(x,expr_at([1,1],before)),
                freeof(x,expr_at([2],before)),
                ident(expr_at([1,1],before),expr_at([2,1],after)),
                ident(expr_at([1,2],before),expr_at([1],after)),
                ident(expr_at([2],before),expr_at([2,2],after)),
                unrelated(sym_at([1],before),sym([2],after)) ] ).

example(isol8, [multocc(x,expr_at([1,2],before)),
                freeof(x,expr_at([1,1],before)),
                freeof(x,expr_at([2],before)),
                ident(expr_at([1,1],before),expr_at([2,1],after)),
                ident(expr_at([1,2],before),expr_at([1],after)),
                ident(expr_at([2],before),expr_at([2,2],after)),
                inverse(sym_at([1],before),sym([2],after)) ] ).
```

```
Utilities package
Prolog-10  version 3

| ?- [winst1,isol].

winst1 consulted   2230 words      0.90 sec.

isol consulted   1610 words      0.45 sec.

yes
| ?- winston(isol).
Please give me an example of a isol
|: isol1.

Please give me an example or near miss of a isol.
|: isol2.

Is this example (yes./no.)?
|: no.

This limits my idea of isol.
Please give me an example or near miss of a isol.
|: isol3.

Is this example (yes./no.)?
|: no.

This limits my idea of isol.
Please give me an example or near miss of a isol.
|: isol4.

Is this example (yes./no.)?
|: no.

This limits my idea of isol.
Please give me an example or near miss of a isol.
|: isol5.

Is this example (yes./no.)?
|: no.

This limits my idea of isol.
Please give me an example or near miss of a isol.
|: isol6.

Is this example (yes./no.)?
|: no.

[ Execution aborted ]
| ?- grey(isol).
Grey areas in isol are:
define([plato1,expr_at([1,2],plato2)],occurtree,[],[2,1])
define([expr_at([2],plato2),expr_at([2,2],plato3)],simtree,[],[2])
define([sym_at([1],plato2),sym([2],plato3)],simtree,[],[1,2])

yes
| ?- winston1(isol).
Please give me an example or near miss of a isol.
|: isol6.
```

```
Is this example (yes./no.)?
|: no.

This limits my idea of isol.
Please give me an example or near miss of a isol.
|: isol7.

Is this example (yes./no.)?
|: no.

This limits my idea of isol.
Please give me an example or near miss of a isol.
|: isol8.

Is this example (yes./no.)?
|: no.

This limits my idea of isol.
I have learnt the concept of isol now.

yes
| ?- grey(isol).
Grey areas in isol are:

yes
| ?- core     51712  (23552 lo-seg + 28160 hi-seg)
heap     18432 =  14955 in use +   3477 free
global    1218 =     16 in use +   1202 free
local     1024 =     16 in use +   1008 free
trail      511 =      0 in use +    511 free
   22.49 sec. for 25 GCs gaining 46683 words
    1.98 sec. for 12 local shifts and 110 trail shifts
   38.49 sec. runtime
```

```
/*block
Winston block domain - simple test example
Alan Bundy 6.12.80
use with winston */


/* space of description tree(s) */
space(block,[shapetree]).

/* description tree(s) */
tree(shapetree,1,shape(prism(wedge,block),pyramid)).

/* Example and near miss */
example(block1,[block(a)]).

example(wedge1,[wedge(b)]).
```

```
/*starch                     It is difficult for the untrained fisher
Winston arch domain          to follow examples of this complexity so
Alan Bundy 5.12.80           here is a simple concept: two wedges.
use with winston */

/* space of description trees */
space(pair,      %  each concept must have a space; is this **right**?
        [shapetree,touchtree,orienttree]).


/* description tree */

tree(shapetree,1,shape(wedge,block)).

tree(touchtree,2,touchrel(separate,touch)).

tree(orienttree,1,orientation(lying,standing)).


%   Examples

example(p1, [wedge(a1), wedge(b1),
        standing(a1), lying(b1), separate(b1, a1)
        ]).

example(p2, [wedge(a2), wedge(b2),
        standing(a2), standing(b2), touch(a2,b2)
        ]).

example(p3, [wedge(a3), wedge(b3),
        lying(a3), lying(b3)
        ]).

%   Near misses

example(n1, [block(a4), block(b4),
        standing(a4), lying(b4), separate(b4, a4)
        ]).     %  two similar things, but not wedges

example(n2, [wedge(a5), wedge(b5), wedge(c5),
        standing(a5), standing(b5), touch(a5,c5)
        ]).     %  one wedge too many

example(n3, [wedge(a6),
        standing(a6)
        ]).     %  one wedge too few
```

page ≐1.

DEPARTMENT OF ARTIFICIAL INTELLIGENCE
PROLOG PROGRAM LIBRARY
PROGRAM SPECIFICATION

Program Name The Winston-Plotkin-Young-Linz Learning Program

Source Alan Bundy

Date of Issue 8 May 1981

1. Description

This is a rational reconstruction of Winston's program, [Winston 75], for learning new concepts, e.g. the arch.[1] It takes descriptions of specimens, hich can be either examples of arches or near misses to arches, and uses them to refine its definition of an arch. The rational reconstructing was done by Plotkin, Young and Linz, as reported (all too briefly) in [Young et al 77]. Their essential advance over Winston was to keep two defining descriptions around: an upper and lower bound; and use incoming evidence to try to move these descriptions closer together.

2. Method of Use

To use the program, type

```
run PLL: UTIL
```

and in response to the prompt type

```
consult('PLL: WINST').
```

The top level predicate, **winston**, will then be available. **winston** takes one argument: the name of the concept to be learnt, e.g. arch. If you call

---

[1] In this program specification we will use 'arch' as the running example. Readers may safely generalize 'arch' to 'concept', wherever it appears, except when indicated to the contrary.

```
winston(arch)
```

then the program will prompt you for the name of a particular arch. It will use this to initialize its lower bound arch description: the upper one being initialized to the contentless description. The program will prompt you for the name of either an arch or a near miss. It will then ask you whether this is an example, to which you must reply either 'yes' or 'no'. All replies to prompts must be terminated with full stop, carriage return.

The program will continue to prompt you with requests for examples or near-misses, until its upper and lower bound descriptions coincide at which point it will announce that it has learnt the concept and will exit **winston**.

If the evidence you provide is already known to the program then it will say so. If it thinks you have provided contradictory evidence then it will say so, try to back up to remake some choice, and then collapse in a heap.

To make the program work you must have compiled the following information:

- A definition of the description space of the concept you want learnt.
- Descriptions of each of the examples and near misses to be input to the program.

The information required for the concept 'arch' can be found in file PLL: ARCH.PRB. It can be input by typing

```
consult('PLL: ARCH.PRB').
```

The description of a specimen is given by the predicate, **specimen**. This predicate takes two arguments: the name of the specimen; and a list of defining propositions, e.g.

```
specimen(arch1, [block(a), block(b), block(c),
        standing(a), standing(b), lying(c),
        leftof(a,b),
        supports(a,c), supports(b,c),
        marries(a,c), marries(c,a), marries(b,c), marries(c,b)]).
```

The predicates used in these descriptions must be arranged into trees of related predicates, and these trees described with the predicate, **tree**. **tree** takes three arguments: the name of the tree; the common arity of all the predicates in it; and the tree itself, represented as a nested term of predicates, e.g.

```
tree(touchtree,2,touchrel(separate,touch(marries,abuts))).
```

This tree is diagrammed in figure 3-1. **touchrel** is a contentless predicate. Two objects may either touch or be separate. Two touching objects may either marry or abut. One of these predicates can be marked as a default with the binary predicate, **default**, e.g.

```
default(touchtree,separate).
```

Finally a list of those predicate trees, which can be used in defining the concept, must be given. This is done with the binary predicate, **space**, which takes the name of the concept and the list of allowed trees, e.g.

```
space(arch,[shapetree,touchtree,orienttree,directiontree,supporttree]).
```

3. How it Works

We first describe the data-structures used by the program and then give an overview of the program.

3.1. Data-Structures

The program's description of a concept consists of a set of predicate trees, with two pointers into each tree: one representing an upper bound; and one a lower bound. Each relation, in the incoming specimen description, is translated into a predicate tree, with a pointer to the relation's predicate (see figure 3-1).

- If every specimen pointer is below the lower bound then the specimen is known to be an example.
- If one specimen pointer is above the upper bound then the specimen is known not to be an example, but to be a near-miss.
- Otherwise, one specimen pointer must lie in the grey area between the upper and lower bound and the program does not already know the status of the specimen. On being told the status, it can modify its definition, by either raising its lower bound to include the specimen pointer (e.g. to predicate 'touch' in figure 3-1), or lowering its upper bound to exclude the specimen pointer (e.g. to predicate 'marries' in figure 3-1).

Note that there will be different predicate trees for different combinations of arguments to the same predicate, e.g. for abuts(a,c), abuts(a,b) and abuts(c,a), say.

A set of predicate trees is represented by a list of terms: each term representing a predicate tree together with pointers. For instance, in the case of the definition of a concept, a typical term might be

```
define([plato1,plato3], touchtree, [], [2,1])
```

define is a four argument function: the second argument is the name of the tree; the first gives the combination of arguments received by the predicates of this tree; and the third and fourth give the positions of the upper and lower bounds, respectively. The constants used as arguments in a concept definition are always called platoN, because they are ideal objects. Positions in the tree are given by lists of positive integers which specify which arcs to follow to traverse the tree from the root to the predicate being pointed to. The empty list specifies the root. The example above corresponds to the situation in figure 3-1.

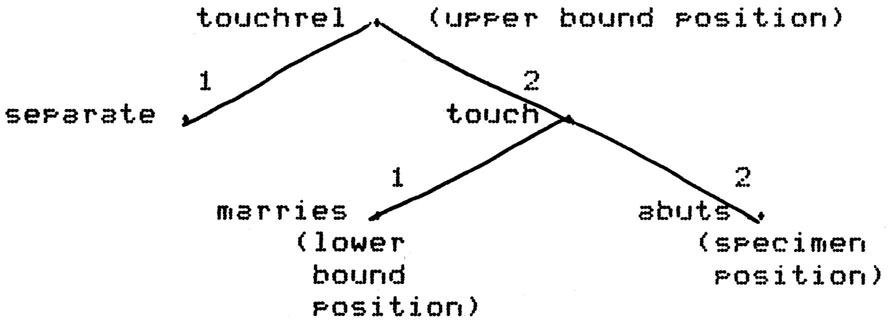


Figure 3-1: A Predicate Tree with Pointers

The description of a specimen is recorded in a similar fashion. The term in this case is constructed from a ternary function, **record**, e.g.

```
record([a,c], touchtree, [2,2])
```

where a and c are constants mentioned in the original specimen description.

Once a match has been established between the (platonic) constants of the definition and the particular constants of the specimen, e.g. {a/plato1, b/plato3, c/plato3}, then a difference description is built up. This is a also a list of terms, but constructed from the six argument function, **differ**, e.g.

```
differ([plato1, plato3], touchtree, [], [2,2], [2,1], grey)
```

This contains, not only, the information from the **record** and **define** terms, but also the status of the specimen, e.g. grey.

3.2. Program Overview

The program is divided into four parts.

1. Top level input/output procedures. The very top level procedure is **winston**, described above, but the heart of the program is the procedure **learn**, which links together the remaining three parts of the program.

   ```
   learn(Concept, Specimen, YesNo) :- !,
           definition(Concept,CObjs,CDefn),
           make_rec(Concept,Specimen,EObjs,ERec),
           classify(CObjs,EObjs,CDefn,ERec,Diff,Verdict),
           learn1(Concept,Diff,YesNo,Verdict).
   ```

   learn takes three input arguments: the concept to be learnt; the current specimen; and its status according to the user. **learn** modifys the program's concept definition appropriately.

2. Procedures to translate the original input descriptions into the internal representation as a set of predicate trees with pointers. The top level procedure of this part is **make_rec**, which takes the concept and the specimen and returns the internal description of the latter as a list of constants and a list of predicate tree records.

3. Procedures to match the constants in the specimen description against those in the stored definition and to classify the resulting description as example, near miss or grey. The top level procedure of this part is **classify**, which takes the constants and predicate trees from both the concept definition and the specimen, and forms, first a difference description and then a classification of the specimen's status.

4. Procedures to act appropriately to this classification, in particular, to adjust the stored definition of the concept when the incoming specimen is classified as 'grey'. The top level procedure of this part is **learn1**, which takes the concept, difference description and the status of the specimen according to both the user and the program.

The best match between the incoming specimen and the definition is found in a crude heuristic way. The heuristic is that even non-examples (near misses) will be almost examples. All matches of objects are tried. For each assignment all corresponding predicate trees are compared. A score for the assignment is totted up: each pair of predicate trees contributing either 0, 2 or 1, according to whether the specimen pointer appears below the lower bound, above the upper bound or in between. The assignment with the lowest score wins.

It can happen that the concept definition contains a predicate tree which does not correspond to any tree in the specimen description, or vice versa. In such cases the program assumes that the missing tree is provided with a pointer to the default predicate in the case of a missing lower bound or specimen position, and a pointer to the tree root in the case of a missing upper bound.

4. Program Requirements

The Prolog system requires 30K words. PLL: UTIL, PLL:WINST, PLL: ARCH.PRB and working space require a further 26K words. The following predicates are used by the program:

add_ups(2)
append(3)
apply(2)
best(2)
checklist(2)
classify(6
common(3)
compare(4)
consts_in(2)
convert(2)
default(2)
default_posn(2)
definition(3)
diff_to_defn(2)
exclude(2)
extra_rec(2)
findall(3)
flatten(2)
gensym(2)
gensym1(3)
grey(1)
grey1(1)
learn(3)
learn1(4)
lowest(4)
lub(2)
make_diff(5)
make_rec(4)
make_subst(3)
maplist(3)
new_defn(2)
nth_el(3)
one_of(3)
pair_off(3)
perm(2)
position(3)
same(1)
score(2)
score1(2)
select(3)
some(2)
space(2)
specimen(2)
subst(3)
sumlist(2)
tree(3)
union(3)
verdict(2)
verdict1(2)
winston(1)
winston1(1)
writef(2)

REFERENCES

[Winston 75]

Winston, P.
Learning structural descriptions from examples.
McGraw Hill, 1975, .

[Young et al 77]

Young, R.M., Plotkin, G.D. and Linz, R.F.
Analysis of an extended concept-learning task.
In Reddy, R., editor, IJCAI-77, pages p285. International Joint Conference on Artificial Intelligence, 1977.

Master

Note 79

Alan Bundy
23 April 1981

LANGUAGE ACQUISTION USING THE WINSTON... PROGRAM

In this note I want to explore the application of the Winston... program to the language acquisition problem studied by Pat Langley with his AMBER program. In particular, I want to address the question as to whether the Winston... technique can be used to obtain the discrimination process used by AMBER.

1. The Description Space

Using the Winston... technique, means building, as a concept definition, the condition part of a production rule like,

action & asing & present & process -> is
or
agent & aplural -> s

where the first rule means

> Use 'is' as a prefix when describing an action and when the agent is singular and the action is in the present and is a process.

and the second rule means

> Use 's' as a suffix, when describing an agent and when the agent is plural.

We will use a description space consisting of 7 relation trees. All the relations will be nullary, although in a pukka version of the program a nullary relation like 'asing' might be translated into agent(X) & singular(X), etc. The trees are all shallow, one level deep, with two or three arcs. They are represented in figure 1-1.

2. Specimen Induction

The first modification required to Winston... is to get it to induce its own specimens* by the following technique.

1. The current state of the concept definition will be used to generate the condition of the rule.

2. This rule will be used to generate the child's utterance.

3. The context of the utterance will be used to get the specimen description.

4. The adult utterance will be used to classify the specimen as an example or near or far miss.

We will assume a mechanism, like AMBER's, to do step 2 and concentrate on steps 1. and 4. (step 3 is trivial).

---

*Where a specimen is either an example, a near miss or a far miss.

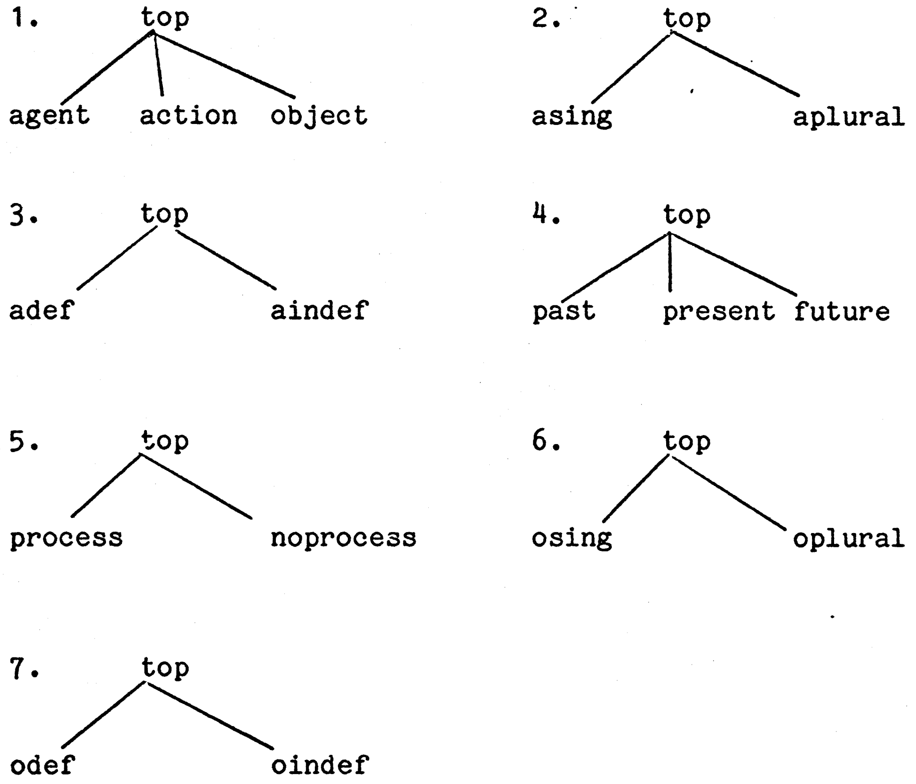


Figure 1-1: Description Space for Language Acquistion

Consider step 1: the use of the current concept definition to generate the rule condition. Each relation tree in the definition will give rise to a single condition. Which node of the tree should be used? In his comparison of Winston and AMBER, Pat assumed that the lower bound would be used in each case, but this is not the only possibility. One could use any node between the upper and lower bound. We will consider two cases: using all the lower bounds and using all the upper bounds. We will see that using all upper bounds leads to a discrimination type process, very like AMBER's.

3. Conditions Formed From Upper Bounds

Consider this case first: that the upper bound of each tree contributes a condition. The initial position of the upper bound in each tree is the root node, marked 'top'. The lower bounds need to be initialized by an example. Let us suppose the child hears an adult use 'is' in the context

action & asing & adef & present & process & oplural & odef

these relations will then be adopted as the initial lower bounds. The condition of the rule generated by the current definition consists of 7 'tops'. Since 'top' is a contentless relation we will omit occurrences of it, so that the current rule is

-> is

The child will now apply the rule in some situation, listen to the adult utterance in this situation and, hence, classify the rule application as being correct, an error of commmission or an error of omission.* The regular cases are summarised in figure 3-1.

```
Correct - Do Nothing
-------------------------- Upper Boundary
Commission - Miss
-------------------------- Target Point
Correct - Example
-------------------------- Lower Boundary
Correct - Example (but no action)
```

Figure 3-1: Cases that Arise when Upper Bounds are Used

Suppose that an error of commission is detected and that the context is

agent & asing & adef & present & process & oplural & odef

This context becomes the description of a near-miss specimen. This is a near-miss, rather than a far-miss, because there is only one grey relation, i.e. only one relation, 'agent', lies in the grey area of the current concept definition. The concept definition can now be updated by moving the upper bound of the first relation tree down from 'top' to 'action'. The rule generated by the upper bounds of the current concept description will contain a non-trivial condition, namely 'action', i.e. it is

action -> is

Suppose that the error of commission was created in a far-miss context, that is, there is more than one grey relation, e.g.

agent & asing & aindef & present & process & oplural & odef

where both 'agent' and 'aindef' are grey relations. The present version of the Winston... program would pick one of them at random and use this as the discrimination relation, i.e. move the upper bound of its relation tree down. Here is an important difference from AMBER. If AMBER has not yet got a major relation (agent, action or object) in its rule condition then it tries to pick a major, grey relation as discrimination relation. In this case, then, AMBER would choose 'agent'. If the condition already includes a major relation or if the context contains no major grey relation, then AMBER forms a new rule for each grey relation. It might be possible to modify the Winston.. program to do things the AMBER way, although it means keeping a whole set of concept definitions around.

If Winston... makes a bad choice when picking the discrimination relation in a far-miss then the resulting concept definition will generate faulty rules: rules that can cause errors of omission as well as errors of commission. We will see that an error of omission indicates that a bad choice has been made

---

*Actually, omission errors cannot happen this time, but they can if the upper bounds get screwed up.

and that the program should back-up and remake the choice.* So let us suppose that in the example above that the program chooses the grey relation 'aindef' and moves the upper bound of the third tree down to 'adef'. This concept definition will generate the rule

adef -> is

Suppose that this rule is used by the child and causes an error of omission, in the context

action & asing & aindef & present & process & oplural & odef

This context constitutes an example description, but one which lies in the non-example part of the current concept defiintion. The Winston... program will back-up, remake the choice of discrimination relation (rejecting 'aindef' and picking 'agent') and then treat the new context as an example, moving the lower bound of the third tree up from 'adef' to 'top'.* This constitutes a difference from AMBER: an erroneous rule has been erased, whereas AMBER keeps such rules around, but with a low strength.

The final case to consider is when the rule has been used correctly. There are two subcases: when the rule has correctly not fired and when it correctly has fired. We only do anything in the second case. For instance, if the rule has correctly fired and the context is

action & asing & aindef & present & process & osing & oindef

This context constitutes an example description. Each grey relation, osing and oindef, will cause the lower bound in the corresponding tree to be lifted (to 'top' in each case). Note that, if this example had come earlier it would have merged the upper and lower bounds of the third tree and prevented the erroneous choice of 'aindef' as the discrimination relation. This is an improvement over AMBER.

4. Conditions Formed From Lower Bounds

Now we consider what happens if the lower bounds are used to generate rule conditions. This is the process implied by Pat in his contrast of the Winston generalization technique and his discrimination technique. We must consider two cases; according to whether the rule is used correctly or causes an error of omission.** These cases are summarised in figure 4-1.

Suppose the initial example is as before then the rule generated from the lower bounds is

---

*In terms of figure 3-1 the target point has been moved up above the upper boundary.

*At least, it should, but this part of the program has never been tested.

**Errors of commission cannot happen, since conditions generated from the lower bounds are always conservative and, unlike the upper bounds, can never be erroneous.

Correct - Miss (but no action)
-------------------------- Upper Boundary
Correct - Miss
-------------------------- Target Point
Omission - Example
-------------------------- Lower Boundary
Correct - Do Nothing

Figure 4-1: Cases that Arise when Lower Bounds are Used

action & asing & adef & present & process & oplural & odef -> is

We consider first the case when the rule is applied correctly. If the rule fired correctly then there is nothing further to do. However, if the rule was not fired, and should not have been, then the new context may still serve as a miss. Suppose the new context is

agent & asing & aindef & present & process & oplural & odef

This contains two grey relations, 'agent' and 'aindef', one of which must be chosen as the discrimination relation. Let us suppose that 'aindef' is incorrectly chosen and the upper bound of the third tree is moved down to merge with the lower bound at 'adef'. The rule condition will be unchanged, but behind the scenes the third tree has been firmed up.* The context of an error of omission serves as an example and if the upper bounds are erroneous (as above) then it may first cause back-up. We will consider only the first case (because it includes the second): i.e. the rule has not fired, but should have, and the context is

action & asing & aindef & present & process & oplural & odef

The program will back-up; remake the choice of discrimination relation (from 'aindef' to 'agent'); restore the third tree; bring down the upper bound of the first tree to merge with the lower bound at 'action'; and then use the above context as an example to raise the lower bound of the third tree to 'top'.

5. Summary

In the last two sections we have considered 6 cases: that conditions are formed solely from the upper bounds or lower bounds and that in each case the rule use is correct or leads to an error of either commission or omission. We summarise the results in table 5-1.

	Correct	Commission	Omission
Upper Bound	Do Nothing or Example	Miss (Near or Far)	Example, *will* cause Back-up
Lower Bound	Do Nothing or Miss	Cannot Happen	Example, may cause Back-up

Table 5-1: Summary of Action on the Six Cases

---

*Erroneously, of course - the target point of figure 4-1 has been moved above the upper boundary.

If rule conditions are generated from the lower bounds of the current concept then the Winston... program behaves much as Pat Langley predicted it would: starting with long, underestimating conditions and settling the rules with long conditions quicker than those with short conditions. If rule conditions are generated from the upper bounds of the current concept, however, then the Winston... program behaves much as Pat's own AMBER program: starting with short, overestimating conditions and settling the rules with short conditions quicker than those with long conditions. Thus the rationally reconstructed Winston... program unifies the old Winston generalization technique and Langley's discrimination technique. Presumably, generating rule conditions from a hybrid of upper and lower bounds would produce a hybrid generalization/discrimination technique, and this possibility might be worth exploring.

Note that a rule's conditions may not change during several episodes, even though the underlying concept definition is changing. Thus, if the conditions are being generated from the upper bounds and some examples come in causing changes to the lower bounds then the conditions will not (cannot) reflect this change. Hence, if the upper bounds are being used and the condition is short, then the condition may become settled early on, subsequent episodes appearing not to change anything, however, these episodes will firm up the conditions by moving the lower bounds up to merge with the upper bounds. If all these episodes are counted in the story of rule formation then rules with short conditions and those with long conditions will take similar lengths of time, despite appearances to the contrary. Dual remarks hold when lower bounds are used to generate rule conditions.

fork

tree (rangetree, 2, rangerel(outrange, inrange (attacks, defends))) .

~~tree (colourtree, 1, colourrel (black, white))~~ stet

~~tree (sidetree, 2, siderel (same, opposite)~~) .

space (fork, [rangetree, ~~sidetree~~ colourrel])

example (fork1, [attacks(wr, bb), attacks(wr, bk), ~~opposite~~ (wr, bb),
opposite(wr, bk), ~~opponent (bb wr) oppont (bk, wr)~~
~~same (bb, bk) same (bk, bb)])~~
white (wr) , black (br) black (bb)].)

for w ⟷ b

(near miss)

~~overload 2~~

exampl overload1 [attacks (br, bb) , defends (br, bk) ,
white (wb) , black (br), black (bb)]

~~attack~~ threat1 [attack (wr,

Old Version.

```
/*Winston
Rational Reconstruction of Winston Learning Program
Alan Bundy 1.12.80*/

/* Top Level Program - learn new concept */
/* ----------------------------------------- */

/*First time only accept an example */
winston(Concept) :- !,
        writef('Please give me an example of a %t \n', [Concept]),
        read(Ex), nl,
        make_rec(Concept,Ex,EObjs,ERec),
        maplist(gensym1(plato),EObjs,CObjs),
        make_subst(EObjs,CObjs,Subst),
        subst(Subst,ERec,CRec),
        maplist(add_ups,CRec,CDefn),
        assert(definition(Concept,CObjs,CDefn)),
        winston1(Concept).

/* Is grey area in definition eliminated? */
winston1(Concept) :-
        definition(Concept,CObjs,CDefn),
        checklist(same,CDefn), !,
        writef('I have learnt the concept of %t now. \n', [Concept]).

/*Subsequently accept either examples or near misses */
winston1(Concept) :- !,
        writef('Please give me an example or near miss of a %t. \n', [Concept]),
        read(Ex), nl,
        writef('Is this example (yes./no.)? \n',[]),
        read(YesNo), nl,
        learn(Concept,Ex,YesNo),
        winston1(Concept).

/* add default upper bounds in concept record */
add_ups(record(Args,Name,Posn), define(Args,Name,[],Posn)).

/* Find position of default predicate on tree */
default_posn(TreeName,Posn) :-
        default(TreeName,Pred), !,
        tree(TreeName,_,Tree), position(Pred,Tree,Posn).

default_posn(TreeName,[]).


/* slight modify to gensym, so it can be used in maplist */
gensym1(Prefix,_,NewConst) :- !, gensym(Prefix,NewConst).


/* are upper and lower bound of concept definition the same? */
same(define(Args,Name,Posn,Posn)).

/* learn from this example or near miss */
learn(Concept, Example, YesNo) :- !,
        definition(Concept,CObjs,CDefn),
        make_rec(Concept,Example,EObjs,ERec),
        classify(CObjs,EObjs,CDefn,ERec,Diff,Verdict),
        learn1(Concept,Diff,YesNo,Verdict).
```

```
/* Make  records from list of relations */
/* ------------------------------------- */

make_rec(Concept,Example,EObjs,ERec) :- !,
        space(Concept,TreeList), example(Example,Relns),
        objs_in(Relns,EObjs),
        form_records(TreeList,Relns,EObjs,ERec).

/* Find all objects mentioned in relations */
objs_in([],[]).
objs_in([Reln|Relns], Objs) :- !,
        Reln =.. [Pred|Args],
        objs_in(Relns,Objs1), union(Args,Objs1,Objs).

/* Change each relation into a record */
form_records([],[],EObjs,[]).

form_records([TreeName|TreeList], Relns1, Objs, Recs) :- !,
        tree(TreeName,Arity,Tree),
        findall(Perm,perm(Objs,Arity,Perm),Perms),
        form_record(TreeName,Relns1,Perms,Recs1,Relns2),
        form_records(TreeList,Relns2,Objs,Recs2),
        append(Recs1,Recs2,Recs).

/* Change the relations relevant to a tree into definitions */
form_record(Name,Relns,[],[],Relns).

/* relation of appropriate type is in list, so convert it */
form_record(Name,Relns1,[Perm|Perms],[record(Perm,Name,Posn)|Rec],Relns3) :-
        tree(Name,Arity,Tree),
        position(Pred,Tree,Posn), Reln =.. [Pred|Perm],
        select(Reln,Relns1,Relns2), !,
        form_record(Name,Relns2,Perms,Rec,Relns3).

/* there is no relation so insert default position */
form_record(Name,Relns1,[Perm|Perms],[record(Perm,Name,Default)|Rec],Relns2):-
        form_record(Name,Relns1,Perms,Rec,Relns2),
        default_posn(Name,Default).

/* find perm of n elements of list */
perm(List,0,[]).

perm([Hd|Tl],N,Perm) :- N>0,
        PN is N - 1,
        perm(Tl,PN,Perm1),
        append(Front,Back,Perm1), append(Front,[Hd|Back],Perm).

perm([Hd|Tl],N,Perm) :-
        N>0, length(Tl,M), M>=N,
        perm(Tl,N,Perm).

/* Find Position of Node in Tree */
position(Node,Tree,[]) :-
        Tree =.. [Node|SubTrees].

position(Node,Tree,[N|Posn]) :-
        Tree =.. [Root|SubTrees],
        nth_el(N,SubTrees,SubTree),
        position(Node,SubTree,Posn).
```

```
/*find nth element of list */
nth_el(1,[Hd|Tl],Hd).

nth_el(N,[Hd|Tl],El) :-
        nth_el(PN,Tl,El), N is PN + 1.

/* Is this example, non-example or in grey area, by my definition? */
/* -------------------------------------------------------------- */

classify(CObjs,EObjs,CDefn,ERec,BestDiff,Verdict) :- !,
        findall(Diff, make_diff(CObjs,EObjs,CDefn,ERec,Diff), Diffs),
        best(Diffs,BestDiff),
        verdict(BestDiff,Verdict).

/* Find the difference between example and concept */
make_diff(CObjs,EObjs,CDefn,ERec,Diff) :- !,
        perm(EObjs,EObjs1), make_subst(EObjs1,CObjs,Subst),
        subst(Subst,ERec,ERec1),
        pair_off(CDefn,ERec1,Diff).

/*Pair off concept definition and example record to make differences */
 air_off([],[],[]) :- !.

pair_off([define(Args,Name,UpPosn,LowPosn) | CDefn],
         ERec,
         [differ(Args,Name,UpPosn,ExPosn,LowPosn,Verdict) | Diff]) :-
                select(record(Args,Name,ExPosn),ERec,Rest), !,
                compare(UpPosn,ExPosn,LowPosn,Verdict),
                pair_off(CDefn,Rest,Diff).

pair_off(CDefn,ERec,Diff) :-
        writef('Error: defn %1 and erec %1 do not match \n',[CDefn,ERec]),
        abort.

/* Compare positions in tree to give verdict */
compare(U,E,L,yes) :- append(L,_,E), !.
compare(U,E,L,grey) :- append(U,_,E), !.
compare(U,E,L,no) :- !.

/* Find best difference and return it */
 est(Diffs,Diff) :- !,
        maplist(score,Diffs,Scores),
        lowest(Diffs,Scores,Diff,Score).

/* Return difference with lowest score */
lowest([Diff],[Score],Diff,Score) :- !.

lowest([Diff1|Diffs], [Score1|Scores], Diff2, Score2) :-
        lowest(Diffs,Scores,Diff2,Score2), Score2<Score1, !.

lowest([Diff|Diffs], [Score|Scores], Diff, Score) :- !.

/* Find score of difference */
score(Diff,Score) :- !,
        maplist(score1,Diff,Scores),
        sumlist(Scores,Score).

/* Find score of individual differ */
score1(differ(_,_,_,_,_,yes), 0) :- !.
score1(differ(_,_,_,_,_,grey), 1) :- !.
```

```
score1(differ(_,_,_,_,_,no), 2) :- !.

/* add up all the numbers in a list */
sumlist([], 0).
sumlist([N|Rest], Total) :- !,
        sumlist(Rest,SubT), Total is SubT + N.

/* Make a substitution for replacing all members of one list
by corresponding members of another list */
make_subst([],[],true).

make_subst([X|XRest],[Y|YRest],X=Y & Subst) :-
        make_subst(XRest,YRest,Subst).


/* Decide whether example falls inside definition on basis of differs */
verdict(Diff,yes) :- checklist(verdict1(yes),Diff), !.
verdict(Diff,no) :- some(verdict1(no),Diff), !.
verdict(Diff,grey) :- some(verdict1(grey),Diff), !.

/* verdict on individual differ */
 erdict1(V, differ(_,_,_,_,_,V)).


/* adjust definition  appropriately */
/* -------------------------------- */

/* if new example found */
learn1(Concept,Diff,yes,grey) :- !,
        writef('This is a new sort of %t. \n', [Concept]),
        maplist(lub,Diff,New),
        retract(definition(Concept,CObjs,Old)),
        assert(definition(Concept,CObjs,New)).

/* if near miss found */
learn1(Concept,Diff,no,grey) :- !,
        writef('This limits my idea of %t. \n', [Concept]),
        one_of(exclude,Diff,Diff1),
        maplist(diff_to_defn,Diff1,New),
        retract(definition(Concept,CObjs,Old)),
        assert(definition(Concept,CObjs,New)).

/* if nothing new is discovered */
learn1(Concept,Diff,Agree,Agree) :- !,
        writef('I have seen one of these before. \n',[]).

/* or if contradiction is discovered */
learn1(Concept,Diff,Agree,Disagree) :- !,
        writef('Uh Oh, somethings gone wrong. I will think again.\n',[]),
        fail.

/* Move lower definition up a bit to include new example */
lub(differ(Args,Name,UpPosn,ExPosn,Old,grey),
    define(Args,Name,UpPosn,New)) :- !,
        common(ExPosn,Old,New).

/* Lower definition already includes new example */
lub(differ(Args,Name,UpPosn,ExPosn,LowPosn,yes),
    define(Args,Name,UpPosn,LowPosn)) :- !.
```

```
/* Move upper definition down a bit to exclude near miss */
exclude(differ(Args,Name,Old,ExPosn,LowPosn,grey),
     differ(Args,Name,New,ExPosn,LowPosn,grey)) :- !,
          common(ExPosn,LowPosn,Comm), append(Comm,[N|_],LowPosn),
          append(Comm,[N],New).

/* Take unnecessary bits out of difference  */
diff_to_defn(differ(Args,Name,UpPosn,ExPosn,LowPosn,Verdict),
               define(Args,Name,UpPosn,LowPosn)).


/* Find common initial sublist of two lists */
common([N|Rest1], [N|Rest2], [N|Rest]) :- !,
          common(Rest1,Rest2,Rest).

common(List1,List2,[]) :- !.

/* change just one member of list */
one_of(Prop, [Old|Tl], [New|Tl]) :- apply(Prop,[Old,New]).
one_of(Prop, [Hd|Old], [Hd|New]) :- one_of(Prop,Old,New).


/* Find out what grey areas still exist in concept */
grey(Concept) :- !,
          writef('Grey areas in %t are:\n',[Concept]),
          definition(Concept,CObjs,CDefn),
          checklist(grey1,CDefn).

grey1(define(Args,Name,Posn,Posn)) :- !.

grey1(Defn) :- !,
          write(Defn), nl.
```

```
                        **********************************
                        * PROLOG CROSS REFERENCE LISTING *
                        **********************************

                                    Unify Xref


   PREDICATE                 FILE           CALLED BY



check_overlap/3          unify          combine/3

combine/3                unify          unify/4

disagree/4               unify          unify/4 find_one/4

find_one/4               unify          disagree/4 find_one/4

free_of/2                unify          make_pair/3

is_variable/1            unify          make_pair/3

make_pair/3              unify          unify/4

mapand/3                 utility        combine/3

memberchk/2              utility        check_overlap/3

occ/3                    utility        free_of/2

subst/3                  utility        unify/4 update/3

unify/3                  unify

unify/4                  unify          unify/3 unify/4

update/3                 unify          <user> combine/3
```

```
/* UNIFY.

Unification procedure for first order logic (with occurs check)
See p80 of Artificial Mathematicians.

Alan Bundy 10.7.81 */


/* Top Level */

unify(Exp1, Exp2, Subst) :-             % To unify two expressions
        unify(Exp1, Exp2, true, Subst). % Start with empty substitution


/* Unify with output and input substitutions */

unify(Exp, Exp, Subst, Subst).          % If expressions are identical, succeedd

unify(Exp1, Exp2, OldSubst, AnsSubst) :-        % otherwise
        disagree(Exp1,Exp2,T1,T2),      % find first disagreement pair
        make_pair(T1,T2,Pair),          % make a substitution out of them, if pos
        combine(Pair,OldSubst,NewSubst),        % combine this with input subst
        subst(Pair,Exp1,NewExp1),       % apply it to expressions
        subst(Pair,Exp2,NewExp2),
        unify(NewExp1,NewExp2,NewSubst,AnsSubst).       % and recurse



/* Find Disagreement Pair */

disagree(Exp1,Exp2,Exp1,Exp2) :-
        Exp1 =.. [Sym1|_], Exp2 =.. [Sym2|_],   % If exps have different
        Sym1 \== Sym2, !.               % function symbol, then succeed

disagree(Exp1,Exp2,T1,T2) :-                            % otherwise
        Exp1 =.. [Sym|Args1], Exp2 =.. [Sym|Args2],     % find their arguments
        find_one(Args1,Args2,T1,T2).                    % and recurse

/* Find first disagreement pair in argument list */

find_one( [Hd | Tl1], [Hd | Tl2],T1,T2) :- !,   % If heads are identical then
        find_one(Tl1,Tl2,T1,T2).                % find disagreement in rest of lis

find_one( [Hd1 | Tl1], [Hd2 | Tl2],T1,T2) :-
        disagree(Hd1,Hd2,T1,T2), !.             % else find it in heads.


/* Try to make substitution out of pair of terms */

make_pair(T1,T2,T1=T2) :-       % T1=T2 is a suitable substitution
        is_variable(T1),        % if T1 is a variable and
        free_of(T1,T2).         % T2 is free of T1

make_pair(T1,T2,T2=T1) :-
        is_variable(T2),        % or if T2 is a variable and
        free_of(T2,T1).         % T1 is free of T2

/* By convention: x,y,z,u,v and w are the only variables */
is_variable(u).         is_variable(v).         is_variable(w).
is_variable(x).         is_variable(y).         is_variable(z).
```

```
/* T is free of X */
free_of(X,T) :- occ(X,T,0).     % if X occurs 0 times in T


/* Combine new substitution pair with old substitution */

combine(Pair, OldSubst, NewSubst) :-
        mapand(update(Pair),OldSubst,Subst1),   % apply new pair to old subst
        check_overlap(Pair,Subst1,NewSubst).    % and delete ambiguous assignments

/* Apply new pair to old substitution */
update(Pair, Y=S, Y=S1) :-              % apply new pair to rhs of old subst
        subst(Pair,S,S1).

/* If X is bound to something already then ignore it */
check_overlap(X=T,Subst,Subst) :-       % Ignore X=T
        memberchk(X=S,Subst), !.        % if there already is an X=S

check_overlap(Pair,Subst,Pair & Subst). % otherwise don't


/* MINI-PROJECTS

1.  Simplify unify to a one way matcher, as per p76 of 'Artificial
Mathematicians'.

2.  Generalize Unify to a simultaneous unifier of a set of expressions,
(gen_unify) as per p80 of 'Artificial Mathematicians'.

3.  Build the associativity axiom into unify (assoc_unify), as per p82
of 'Artificial Mathematicians'.

*/
```

```
/* SIMPLE.

Using PROLOG as a theorem prover:
An equality axioms example.
Try goal 'equal(y,x).'.

Alan Bundy  16.6.81 */


equal(x,y).                 % The Hypothesis

equal(X,X).                 % The Reflexive Axiom

equal(U,W) :- equal(U,V), equal(W,V).   % The Twisted Transitivity Axiom


/* MINI-PROJECTS

1. Try goal ?- equal(y,x).

2  Experiment by switching the order of the above axioms and trying
the same goal.  What sort of bad behaviour emerges?  How could it be
avoided?

3. Experiment with different axioms, e.g. the group theory example
from p92 of 'Artificial Mathematicians'.

*/
```

```
/* EQUAL.

Clauses for the SIMPLE equality example
Symmetry can be inferred from reflexivity and twisted transitivity

Alan Bundy  22.6.81 */

clause(hypothesis, [equal(x,y)], [], input ).   % Input clauses
clause(reflexive,  [equal(X,X)], [], input ).
clause(twisted,    [equal(U,W)], [equal(U,V), equal(W,V)], input ).

clause(goal,       [], [equal(y,x)], topclause).        % Top clause
```

```
                     **********************************
                     * PROLOG CROSS REFERENCE LISTING *
                     **********************************


                           Equal and Breadt Xref


  PREDICATE                 FILE            CALLED BY



append/3                  utility        resolve/3

breadth/1                 breadt         go/0 breadth/1

clause/4                  equal          find_clause/4

find_clause/4             breadt         resolve/3 record_clause/3

gensym/2                  utility        record_clause/3

go/0                      breadt

record_clause/3           breadt         resolve/3

repeat/1                  breadt         breadth/1

resolve/3                 breadt

select/3                  utility        resolve/3

writef/1                  utility        record_clause/3

writef/2                  utility        record_clause/3
```

```
 BREADTH.

 Breadth First Search Theorem Prover.
UAL contains test example.

an Bundy 16.6.81 */


* Top Goal */

) :- breadth(0).


* Breadth First Search Strategy */

readth(N) :-
       N1 is N+1,                       % Calculate new depth
       repeat(resolve(input,N,N1)),     % Form all resolvants at that depth
       repeat(resolve(N,input,N1)),
       breadth(N1).                     % and recurse


* Repeat as many times as possible */

epeat(Goal) :- Goal, fail.      % Keep trying and failing
epeat(Goal).                    % and succeed only when you run out of things to do


* Resolution Step */

esolve( N1, N2, N ) :-
       find_clause(Parent1, Consequent1, Antecedent1, N1),     % Find clauses at
       find_clause(Parent2, Consequent2, Antecedent2, N2),     % appropriate depth
       select(Literal, Consequent1, RestConse1),       % find a common literal
       select(Literal, Antecedent2, RestAnte2),        % return leftovers
       append(RestConse1, Consequent2, Consequent),    % cobble leftovers together
       append(Antecedent1, RestAnte2, Antecedent),
       record_clause(Consequent,Antecedent,N).         % record new clause


* Record Existence of New Clause */
ecord_clause([],[],N) :-                        % test for empty clause
       writef('Success! Empty Clause Found\n\n'), !,    % tell user
       abort.                                   % and stop

ecord_clause(Consequent,Antecedent,N) :-        % test for loop
       find_clause(Name,Consequent,Antecedent,M), !.    % i.e. clause with same inr

ecord_clause(Consequent,Antecedent,N) :- !,     % record new clause
       gensym(clause,Name),                     % make up new name
       assert(clause(Name,Consequent,Antecedent,N)),    % assert clause
       writef('%t is name of new resolvant %l <- %l at depth %t\n\n',
               [Name,Consequent,Antecedent,N]),         % tell user


* Find a clause at depth N */

ind_clause(Name,Consequent,Antecedent,0) :-
       clause(Name,Consequent,Antecedent,topclause), !.
```

```
ind_clause(Name,Consequent,Antecedent,N) :-
        clause(Name,Consequent,Antecedent,N).
```

* MINI-PROJECTS

. Try this theorem prover with the clauses of file EQUAL.

. Experiment by making up some clauses of your own and trying them
ut.

. Modify the theorem prover to print out the solution when it has
ound it.

. Modify the theorem prover to remove the input restriction.

. Modify the theorem prover to incorporate the literal selection
estriction.

. Build a depth first theorem prover along the same lines.

```
yes
| ?- breadth(0).
clause1 is name of new resolvant
 <-
    equal(y,_101)
    equal(x,_101)
 at depth 1

clause2 is name of new resolvant
 <-
    equal(y,y)
 at depth 2

clause3 is name of new resolvant
 <-
    equal(x,y)
 at depth 2

clause4 is name of new resolvant
 <-
    equal(y,_128)
    equal(_127,_128)
    equal(x,_127)
 at depth 2

clause5 is name of new resolvant
 <-
    equal(x,_128)
    equal(_127,_128)
    equal(y,_127)
 at depth 2

Success! Empty Clause Found

[ Execution aborted ]

| ?- core     60416  (31232 lo-seg + 29184 hi-seg)
heap     26112 =  23649 in use +   2463 free
 lobal    1177 =     16 in use +   1161 free
local     1024 =     16 in use +   1008 free
trail      511 =      0 in use +    511 free
    0.65 sec. runtime
```

```
                  **********************************
                  * PROLOG CROSS REFERENCE LISTING *
                  **********************************

                        Equal and Heuris Xref


  PREDICATE                FILE           CALLED BY



append/3                 utility        heuristic/1 resolve/3

clause/4                 equal          go/0 successor/2 resolve/3 record_clause/3
                                        pick_best/3 pick_best/5 find_clause/4

  mpare/8                heuris         pick_best/5

evaluate/3               heuris         record_clause/3 find_clause/4

find_clause/4            heuris

findall/3                utility        heuristic/1

gensym/2                 utility        record_clause/3

go/0                     heuris

heuristic/1              heuris         go/0 heuristic/1

pick_best/3              heuris         heuristic/1

pick_best/5              heuris         pick_best/3 pick_best/5

  cord_clause/3          heuris         resolve/3

resolve/3                heuris         successor/2

select/3                 utility        resolve/3

successor/2              heuris         <user> heuristic/1

writef/1                 utility        record_clause/3

writef/2                 utility        record_clause/3
```

```
* HEURIS.

A Heurisitic Search Theorem Prover.
EQUAL contains test example.

Alan Bundy 19.6.81 */


/* Top Goal */

go :-
        clause(Goal,_,_,topclause),
        heuristic([Goal]).


/* Heuristic Search Strategy */

heuristic(Fringe) :-
        pick_best(Fringe,Current,Rest),           % Pick the clause with best score
        findall(Clause,successor(Current,Clause),NewClauses),    % findall its succ
        append(Rest,NewClauses,NewFringe),        % Put them on fringe
        heuristic(NewFringe).                     % and recurse


/* Clause is a resolvant of Current with an input clause */

successor(Current,Clause) :-
        clause(Input,_,_,input),          % Pick an input clause
        ( resolve(Current,Input,Clause) ; resolve(Input,Current,Clause) ).
                                          % resolve it with the current clause


/* Resolution Step */

resolve( Parent1, Parent2, Resolvant) :-
        clause(Parent1, Consequent1, Antecedent1, N1),   % Get the two parents
        clause(Parent2, Consequent2, Antecedent2, N2),
        select(Literal, Consequent1, RestConse1),        % Select a common literal
        select(Literal, Antecedent2, RestAnte2),         % and return the rest
        append(RestConse1, Consequent2, Consequent),     % Join the odd bits togeth
        append(Antecedent1, RestAnte2, Antecedent),
        record_clause(Consequent,Antecedent,Resolvant). % Record the clause


/* Record Existence of New Clause */
record_clause([],[],empty) :-                     % test for empty clause
        writef('Success! Empty Clause Found\n\n'), !, % tell the user
        abort.                                    % and stop

record_clause(Consequent,Antecedent,Name) :-      % test for loop
        clause(Name,Consequent,Antecedent,M), !, fail.

record_clause(Consequent,Antecedent,Name) :- !, % record new clause
        gensym(clause,Name),                      % make up a name
        evaluate(Consequent,Antecedent,N),        % set score of clause
        assert(clause(Name,Consequent,Antecedent,N)),    % assert clause
        writef('%t is name of new resolvant %l <- %l with score %t\n\n',
                [Name,Consequent,Antecedent,N]).         % tell user
```

```
/* Evaluation Function on Clauses (length of clause) */

evaluate(Consequent,Antecedent,Score) :-
        length(Consequent,C),    % add length of rhs
        length(Antecedent,A),    % to length of lhs
        Score is C+A.            % to get clause length


/* Pick clause with best score (i.e.lowest) */

pick_best([Hd|Tl],Choice,Rest) :-
        clause(Hd,_,_,N),                    % Get score of first clause
        pick_best(Tl,Hd,N,Choice,Rest).  % and run down list remembering best so fa

pick_best([],Hd,N,Hd,[]).           % When you get to the end return running score

pick_best([Hd1|Tl1],Hd,N,Choice,[Hd3|Rest]) :-
        clause(Hd1,_,_,N1),                          % Get score of first clause
        compare(Hd,N,Hd1,N1,Hd2,N2,Hd3,N3),      % Compare with running score and o
        pick_best(Tl1,Hd2,N2,Choice,Rest).       % recurse with new best score

compare(Hd,N,Hd1,N1,Hd,N,Hd1,N1) :-       % put running score first
        N =< N1, !.                          % unless new score is best

compare(Hd,N,Hd1,N1,Hd1,N1,Hd,N).        % Otherwise put new score first


/* Find a clause with score N */

find_clause(Name,Consequent,Antecedent,N) :-
        clause(Name,Consequent,Antecedent,topclause), !,
        evaluate(Consequent,Antecedent,N).
```

not called?

```
find_clause(Name,Consequent,Antecedent,N) :-
        clause(Name,Consequent,Antecedent,N).


/* MINI-PROJECTS

1   Try this theorem prover with the equality clauses of EQUAL.

2.  Experiment with clauses of your own devising.

3.  Modify the theorem prover to print out the solution when it has
found it.

4.  Modify the theorem prover to remove the input restriction.

5.  Experiment with different versions of the evaluation function by
modifying 'evaluate' (see section 6.5.3 of 'Artificial
Mathematicians').

*/
```

```
es
 ?- heuristic([goal]).
lause1 is name of new resolvant
<-
    equal(y,_158)
    equal(x,_158)
with score 2

lause2 is name of new resolvant
<-
    equal(y,y)
with score 1

lause3 is name of new resolvant
<-
    equal(x,y)
with score 1

lause4 is name of new resolvant

  ✓ equal(y,_229)
    equal(_228,_229)
    equal(x,_228)
with score 3

lause5 is name of new resolvant
<-
    equal(x,_229)
    equal(_228,_229)
    equal(y,_228)
with score 3

Success! Empty Clause Found

[ Execution aborted ]

| ?- core     60928  (31744 lo-seg + 29184 hi-seg)
heap     26624 =  23909 in use +   2715 free
global    1177 =     16 in use +   1161 free
local     1024 =     16 in use +   1008 free
trail      511 =      0 in use +    511 free
    0.01 sec. for 1 trail shift
    0.80 sec. runtime
```

```
                       **********************************
                       * PROLOG CROSS REFERENCE LISTING *
                       **********************************


                         Semant, Model and Divide Xref


  PREDICATE               FILE           CALLED BY



append/3                utility       resolve/3 paramodulate/3

apply/2                 utility       some/3

checklist/2             utility       model/2

clause/4                divide        go/0 successor/2 resolve/3  paramodulate/3
                                      record_clause/3 model/2

evaluate/3              model         <user> evaluate/3 is_true/2 is_false/2

gensym/2                utility       record_clause/3

go/0                    semant

interpret/3             divide        evaluate/3

interpretation/1        divide        satisfiable/1

is_false/2              model         <user> model/2

is_true/2               model         <user> model/2

  plist/3               utility       evaluate/3

model/2                 model         satisfiable/1

paramodulate/3          semant        successor/2

record_clause/3         semant        resolve/3 paramodulate/3

replace/6               semant        paramodulate/3

replace1/4              semant        replace/6 replace2/4

replace2/4              semant

resolve/3               semant        successor/2

satisfiable/1           model         vet/1
```

```
select/3              utility       resolve/3 paramodulate/3

semantic/1            semant        go/0 semantic/1

some/3                semant        replace1/4 some/3

successor/2           semant        semantic/1

vet/1                 model         semantic/1

writef/1              utility       record_clause/3

writef/2              utility       record_clause/3 satisfiable/1
```

```
/* SEMANT.

A Depth First Theorem Prover
with vetting by use of interpretations and
incorporating input restriction.
Use with MODEL
DIVIDE contains test example

Alan Bundy 22.6.81 */


/* Top Goal */

go :-
        clause(Goal,_,_,topclause),      % Find the top clause
        semantic(Goal).                  % and away you go


/* Depth first theorem prover */
semantic(Old) :-
        successor(Old,New),      % Find a successor to the current clause
        vet(New),                % check that it is unsatisfiable
        semantic(New).           % and recurse

/* Clause is a resolvant of Current with an input clause */

successor(Current,Clause) :-
        clause(Input,_,_,input),                 % Pick an input clause
        ( resolve(Current,Input,Clause) ;        % resolve it with the
          resolve(Input,Current,Clause) ;        % current clause
          paramodulate(Current,Input,Clause) ;   % or paramodulate it
          paramodulate(Input,Current,Clause) ).



/* Resolution Step */

resolve( Parent1, Parent2, Resolvant) :-
        clause(Parent1, Consequent1, Antecedent1, _),   % Get the two parents
        clause(Parent2, Consequent2, Antecedent2, _),
        select(Literal, Consequent1, RestConse1),       % Select a common literal
        select(Literal, Antecedent2, RestAnte2),        % and return the rest
        append(RestConse1, Consequent2, Consequent),    % Join the odd bits togeth
        append(Antecedent1, RestAnte2, Antecedent),
        record_clause(Consequent,Antecedent,Resolvant). % Record the clause



/* Paramodulation Step */

paramodulate( Parent1, Parent2, Paramodulant ) :-
        clause(Parent1, Consequent1, Antecedent1, _),   % Get the two parents
        clause(Parent2, Consequent2, Antecedent2, _),
        select(equal(T,S), Consequent1, RestConse1),    % select an equation
        replace(T,S,Consequent2,Antecedent2,NewConse2,NewAnte2),% put it in the ot
        append(RestConse1,NewConse2,Consequent),        % Join the odd bits togeth
        append(Antecedent1,NewAnte2,Antecedent),
        record_clause(Consequent,Antecedent,Paramodulant).      %record the clause
```

```
/* Replace T by S (or S by T) in clause */

replace(T,S,OldConse,OldAnte,NewConse,OldAnte) :-      % replace T by S in
        replace1(T,S,OldConse,NewConse).                % the consequent

replace(T,S,OldConse,OldAnte,OldConse,NewAnte) :-      % or T by S in
        replace1(T,S,OldAnte,NewAnte).                  % the antecedent

replace(S,T,OldConse,OldAnte,NewConse,OldAnte) :-      % or S by T in
        replace1(S,T,OldConse,NewConse).                % the consequent

replace(S,T,OldConse,OldAnte,OldConse,NewAnte) :-      % or S by T in
        replace1(S,T,OldAnte,NewAnte).                  % the antecedent


/* Replace T by S in Old to get New */

replace1(T,S,Old,New) :-
        some(replace2(T,S),Old,New).    % replace one of the literals

replace2(T,S,T,S).              % replace this occurence

replace2(T,S,Old,New) :-                % replace one of the arguments
        Old =.. [Sym | OldArgs],        % get the arguments
        replace1(T,S,OldArgs,NewArgs),  % replace one
        New =.. [Sym | NewArgs].        % put it all together again



/* Record Existence of New Clause */

record_clause([],[],empty) :-                   % test for empty clause
        writef('Success! Empty Clause Found\n\n'), !,   % tell user
        abort.                                  % and stop

record_clause(Consequent,Antecedent,Name) :-    % test for loop
        clause(Name,Consequent,Antecedent,_), !.        % i.e. clause with same in

record_clause(Consequent,Antecedent,Name) :- !, % record new clause
        gensym(clause,Name),                    % make up new name
        assert(clause(Name,Consequent,Antecedent,new)), % assert clause
        writef('%t is name of new resolvant %l <- %l \n\n',
                [Name,Consequent,Antecedent]).  % tell user


/* Apply Pred to just one element of list */

some(Pred, [Hd1 | Tl], [Hd2 | Tl]) :-
        apply(Pred, [Hd1,Hd2]).         % apply it to this one

some(Pred, [Hd | Tl1], [Hd | Tl2]) :-
        some( Pred, Tl1, Tl2).          % or one of the others


/* MINI-PROJECTS

1.  Try out theorem prover with the arithmetic clauses and models of
file DIVIDE.

2.  Experiment with some clauses and models of your own devising (See
```

chapter 10 for some ideas).

8. Modify the theorem prover so that it works by breadth first search (Compare with file BREADT).

9. Modify the theorem prover so that it works by heuristic search (Compare with file HEURIS).

K/

```
* MODEL.

ow to evaluate a clause in an interpretation
provided it is variable free!!)

lan Bundy 22.6.81 */


* Vet the clause in any interpretations */

et(Clause) :-
        not satisfiable(Clause).        % Clause has no model


* Clause is satisfiable in some Interpretation */

atisfiable(Clause) :-
        interpretation(Interp),         % If there is an interpretation
        model(Interp,Clause),           % in which Clause is true
        writef('%t rejected by %t\n\n',[Clause,Interp]).        % tell user


* Interpretation is a model of a Clause */

odel(Interp, Clause) :-
        clause(Clause,Consequent,Antecedent,_), % Get Clause definition
        checklist(is_true(Interp), Consequent), % Check all lhs literals are true
        checklist(is_false(Interp), Antecedent).        % and all rhs ones are fals


* Evaluate expression in Interpretation */

valuate( Interp, Integer, Integer ) :- integer(Integer), !.    % integers represen

valuate( Interp, Constant, Value ) :-           % other constants have values assig
        atom(Constant), !, interpret(Interp, Constant, Value).

valuate( Interp, Complex, Value ) :-
        Complex =.. [Sym | Args], !,            % recurse on arguments of
        maplist(evaluate(Interp), Args, Vals),  % complex terms
        Complex1 =.. [Sym | Vals],              % then interpret topmost
        interpret(Interp, Complex1, Value).     % symbol


* Evaluation of clause (hacks for checklist) */

s_true(Interp, Literal) :- evaluate(Interp, Literal, true).

s_false(Interp, Literal) :- evaluate(Interp, Literal, false).
```

```
/* DIVIDE.

Clauses and Model for not divides example (see notes p124)

Alan Bundy  22.6.81 */

/* Clauses */

clause(right, [not_div(X*Z,Y)], [not_div(X,Y)], input). % Input clauses

clause(left, [not_div(Z*X,Y)], [not_div(X,Y)], input).

clause(thirty, [equal(30,2*3*5)], [], input).

clause(hypothesis, [not_div(5,a)], [], input).

clause(conclusion, [], [not_div(30,a)], topclause).     % Top clause



/* Models */

/* arith2 */

interpretation(arith2).          % arith2 is an interpretation

interpret(arith2, a, 2).                                % meaning of a

interpret(arith2, not_div(X,Y), false) :-       % meaning of not_div
        0 is Y mod X, !.
interpret(arith2, not_div(X,Y), true).

interpret(arith2, equal(X,Y), true) :-          % meaning of equal
        X =:= Y, !.
interpret(arith2, equal(X,Y), false).

interpret(arith2, X*Y, Z) :- Z is X*Y.          % meaning of *


/* arith3 */

interpretation(arith3).          % arith3 is an interpretation

interpret(arith3, a, 3).                                % meaning of a

interpret(arith3, not_div(X,Y), false) :-       % meaning of not_div
        0 is Y mod X, !.
interpret(arith3, not_div(X,Y), true).

interpret(arith3, equal(X,Y), true) :-          % meaning of equal
        X =:= Y, !.
interpret(arith3, equal(X,Y), false).

interpret(arith3, X*Y, Z) :- Z is X*Y.          % meaning of *
```

```
yes
| ?- go.
clause1 is name of new resolvant
 <-
    not_div(2*3*5,a)


lause2 is name of new resolvant
 <-
    not_div(2*3,a)


lause3 is name of new resolvant
 <-
    not_div(2,a)


lause3 rejected by arith2

ause4 is name of new resolvant
   -
    not_div(3,a)


lause4 rejected by arith3

lause5 is name of new resolvant
 <-
    not_div(5,a)


uccess! Empty Clause Found

 Execution aborted ]

| ?- core     60928  (31744 lo-segs + 29184 hi-segs)
eap      26624 =  24583 in use +   2041 free
global    1177 =     16 in use +   1161 free
cal       1024 =     16 in use +   1008 free
ail        511 =      0 in use +    511 free
    0.10 sec. for 2 GCs gaining 625 words
    0.04 sec. for 3 local shifts and 7 trail shifts
    2.94 sec. runtime
```

```
                         **********************************
                         * PROLOG CROSS REFERENCE LISTING *
                         **********************************

                                    Rewrit Xref


   PREDICATE                FILE              CALLED BY



 apply/2                  utility         some/3

 go/0                     rewrit

 normalize/2              rewrit          normalize/2 go/0

 replace/4                rewrit          rewrite/2

 rewrite/2                rewrit          normalize/2

 rule/3                   rewrit          rewrite/2

 some/3                   rewrit          replace/4 some/3

 writef/2                 utility         rewrite/2 go/0
```

```
* REWRIT.

imple depth first search rewrite rule system
ee Artificial Mathematicians p 104.
se with UTIL

lan Bundy  10.7.81 */


* Find Normal Form of Expression */

ormalize(Expression, NormalForm) :-     % To put an expression in normal form:
        rewrite(Expression, Rewriting),         % Rewrite it once
        normalize(Rewriting, NormalForm).       % and recurse

ormalize(Expression, Expression) :-     % The expression is in normal form
        not rewrite(Expression, _).     % if it cannot be rewritten


* Rewrite Rule of Inference */

ewrite(Expression, Rewriting) :-        % To rewrite Expression
        rule(Name,LHS, RHS),            % get a rule LHS => RHS
        replace(LHS, RHS, Expression, Rewriting),       % replace LHS by RHS
        writef('%t rewritten to %t by rule %t\n', [Expression,Rewriting,Name]).


* Replace single occurrence of T by S in Old to get New */

eplace(T,S,T,S).                % replace this occurence

eplace(T,S,Old,New) :-          % replace one of the arguments
        Old =.. [Sym | OldArgs],        % get the arguments
        some(replace(T,S),OldArgs,NewArgs),     % replace one
        New =.. [Sym | NewArgs].        % put it all together again

* Apply Pred to just one element of list */

ome(Pred, [Hd1 | Tl], [Hd2 | Tl]) :-
        apply(Pred, [Hd1,Hd2]).         % apply it to this one

ome(Pred, [Hd | Tl1], [Hd | Tl2]) :-
        some( Pred, Tl1, Tl2).          % or one of the others


* Some rules */

ule(1,X*0, 0).          % Algebraic Simplification rules
ule(2,1*X, X).
ule(3,X^0, 1).
ule(4,X+0, X).


* A Typical Problem */

o :- normalize( (a^(2*0))*5 + b*0, NormalForm),
        writef('Normal Form is %t\n\n', [NormalForm]),
        fail.
```

```
/* MINI-PROJECTS

. Try out the system with the arithmetic rules given above.

2.  Experiment with some rules of your own devising (Suggestions can be
found in chapter 9 and section 5.2 of 'Artificial Mathematicians').

3.  Modify the system so that it works by:  (a) call by value, (b) call
by name (See p107 of 'Artificial Mathematicians').

*/
```

```
yes
| ?- so.
a^(2*0)*5+b*0 rewritten to a^0*5+b*0 by rule 1
a^0*5+b*0 rewritten to a^0*5+0 by rule 1
a^0*5+0 rewritten to 1*5+0 by rule 3
1*5+0 rewritten to 5+0 by rule 2
5+0 rewritten to 5 by rule 4
Normal Form is 5

5+0 rewritten to 5 by rule 4
1*5+0 rewritten to 1*5 by rule 4
1*5 rewritten to 5 by rule 2
Normal Form is 5

1*5 rewritten to 5 by rule 2
1*5+0 rewritten to 5+0 by rule 2
a^0*5+0 rewritten to a^0*5 by rule 4
a^0*5 rewritten to 1*5 by rule 3
1*5 rewritten to 5 by rule 2
Normal Form is 5

1*5 rewritten to 5 by rule 2
a^0*5 rewritten to 1*5 by rule 3
a^0*5+0 rewritten to 1*5+0 by rule 3
a^0*5+b*0 rewritten to 1*5+b*0 by rule 3
1*5+b*0 rewritten to 1*5+0 by rule 1
1*5+0 rewritten to 5+0 by rule 2
5+0 rewritten to 5 by rule 4
Normal Form is 5

5+0 rewritten to 5 by rule 4
1*5+0 rewritten to 1*5 by rule 4
1*5 rewritten to 5 by rule 2
Normal Form is 5

1*5 rewritten to 5 by rule 2
1*5+0 rewritten to 5+0 by rule 2
1*5+b*0 rewritten to 5+b*0 by rule 2
5+b*0 rewritten to 5+0 by rule 1
5+0 rewritten to 5 by rule 4
Normal Form is 5

5+0 rewritten to 5 by rule 4
5+b*0 rewritten to 5+0 by rule 1
1*5+b*0 rewritten to 1*5+0 by rule 1
a^0*5+b*0 rewritten to a^0*5+0 by rule 1
a^(2*0)*5+b*0 rewritten to a^(2*0)*5+0 by rule 1
a^(2*0)*5+0 rewritten to a^0*5+0 by rule 1
a^0*5+0 rewritten to 1*5+0 by rule 3
1*5+0 rewritten to 5+0 by rule 2
5+0 rewritten to 5 by rule 4
Normal Form is 5

5+0 rewritten to 5 by rule 4
1*5+0 rewritten to 1*5 by rule 4
1*5 rewritten to 5 by rule 2
Normal Form is 5

1*5 rewritten to 5 by rule 2
```

```
.*5+0 rewritten to 5+0 by rule 2
:^0*5+0 rewritten to a^0*5 by rule 4
:^0*5 rewritten to 1*5 by rule 3
.*5 rewritten to 5 by rule 2
lormal Form is 5

.*5 rewritten to 5 by rule 2
:^0*5 rewritten to 1*5 by rule 3
:^0*5+0 rewritten to 1*5+0 by rule 3
:^(2*0)*5+0 rewritten to a^(2*0)*5 by rule 4
:^(2*0)*5 rewritten to a^0*5 by rule 1
:^0*5 rewritten to 1*5 by rule 3
.*5 rewritten to 5 by rule 2
lormal Form is 5

.*5 rewritten to 5 by rule 2
:^0*5 rewritten to 1*5 by rule 3
:^(2*0)*5 rewritten to a^0*5 by rule 1
:^(2*0)*5+0 rewritten to a^0*5+0 by rule 1
:^(2*0)*5+b*0 rewritten to a^0*5+b*0 by rule 1

 ?- core     60416  (31232 lo-seg + 29184 hi-seg)
ıeap      26112 =  23454 in use +   2658 free
:lobal     1175 =     16 in use +   1159 free
.ocal      1024 =     16 in use +   1008 free
:rail       511 =      0 in use +    511 free
    0.00 sec. for 1 trail shift
    3.02 sec. runtime
```

```
                    ***********************************
                    * PROLOG CROSS REFERENCE LISTING *
                    ***********************************

                                Skolem.Xref


  PREDICATE                FILE            CALLED BY



gensym/2                 utility        skolem/4

opposite/2               skolem         skolem/4

skolem/2                 skolem         skolem/4  test1/1  test2/1

skolem/4                 skolem         skolem/2  skolem/4

subst/3                  utility        skolem/4

test1/1                  formul

test2/1                  formul

univ/3                   skolem         skolem/4
```

```
/* SKOLEM.

Skolem Normal Form Procedure
(FORMUL contains test examples)

Alan Bundy 17.3.79 */


/*operator declarations*/

:- op(400,xfy,;).               % Disjunction

:- op(300,xfy,<->).             % Double Implication

:-op(400,xfy,->).               % Implication


/*skolem normal form*/

skolem(Sentence,NormalForm) :- !,               % Normal calling pattern
        skolem(Sentence,NormalForm,[],0).       % assume no free vars and being as


skolem(P <-> Q, (P1->Q1)&(Q1->P1),Vars,Par) :- !,       % Double implication
        skolem(P,P1,Vars,Par), skolem(Q,Q1,Vars,Par).

skolem(all(X,P),P1,Vars,0) :- !,                % Universal quantification
        skolem(P,P1,[X|Vars],0).

skolem(all(X,P),P2,Vars,1) :- !,                % Universal quantification
        gensym(f,Fs), univ(Fs,Vars,F),          % dual case
        subst(X=F,P,P1),
        skolem(P1,P2,Vars,1).

skolem(some(X,P),P2,Vars,0) :- !,               % Existential quantification
        gensym(f,Fs), univ(Fs,Vars,F),
        subst(X=F,P,P1),
        skolem(P1,P2,Vars,0).

skolem(some(X,P),P1,Vars,1) :- !,               % Existential quantification
        skolem(P,P1,[X|Vars],1).                % dual case

skolem(P->Q,P1->Q1,Vars,Par) :- !,                      % Implication
        opposite(Par,Par1), skolem(P,P1,Vars,Par1),
        skolem(Q,Q1,Vars,Par).

skolem(P;Q,P1;Q1,Vars,Par) :- !,                        % Disjunction
        skolem(P,P1,Vars,Par), skolem(Q,Q1,Vars,Par).

skolem(P&Q,P1&Q1,Vars,Par) :- !,                        % Conjunction
        skolem(P,P1,Vars,Par), skolem(Q,Q1,Vars,Par).

skolem(not P,not P1,Vars,Par) :- !,                     % Negation
        opposite(Par,Par1), skolem(P,P1).

skolem(Pred,Pred,Vars,Par) :- !.                % Atomic formula


*opposite parities*/
```

```
opposite(0,1).
opposite(1,0).

/*univ - apply args to symbol*/
univ(Fs,Vars,F) :- !,
        F =.. [Fs|Vars].


/* MINI-PROJECTS

1. Try out on various formulae.

2. Modify the program to deal with bounded quantification, e.g.
all_in(X,Set,P) - meaning, for all X in Set, P is true.

3.  Build programs for putting formulae in:  (a) Prenex Normal Form, (b)
Conjunctive Normal Form, as per section 5.2 of 'Artificial
Mathematicians'. You may wish to use file REWRIT.

*/
```

```
/* FORMUL.

Test Formulae for SKOLEM program

Alan Bundy 23.6.81 */


test1(Ans) :-
        skolem( all(a, all(b, all(c, some(x, a*x^2 + b*x + c = 0)))), Ans).

test2(Ans) :-
        skolem( all(m, some(delta, all(x, (abs(x)=<delta) -> (1/x)>m))), Ans).
```

```
yes
| ?- test1(A).

A = a*f1(c,b,a)^2+b*f1(c,b,a)+c=0

yes
| ?- test2(A).

A = (abs(x)=<f2(m))->1/x>m

yes
| ?- core      60416  (31232 lo-seg + 29184 hi-seg)
heap     26112 =  23809 in use +   2303 free
global    1177 =     16 in use +   1161 free
local     1024 =     16 in use +   1008 free
trail      511 =      0 in use +    511 free
    1.00 sec. runtime
```

```
                    **********************************
                    * PROLOG CROSS REFERENCE LISTING *
                    **********************************


                                Boyer Xref


  PREDICATE                FILE           CALLED BY



add_bomb/3               boyer

analyse/3                boyer         ugliness/3

append/3                 utility       find_ugly/3 add_bomb/3

def_eval/2               boyer         symbol_eval/3 open_eval/2

eval/2                   utility

eval/2                   boyer         symbol_eval/3 def_eval/2

expand/4                 boyer         symbol_eval/3

fertilise/3              boyer         prove_step/2

find_ugly/3              boyer         ugliness/3 find_ugly/3

generalise/2             undefined     prove1/2

loop_threat/2            boyer         analyse/3

make_fault/2             boyer         expand/4 make_fault/2

max/5                    boyer         pickindvars/3 max/5

memberchk/2              utility       nontrivial/1

merge/3                  boyer         symbol_eval/3 rewrite/3 merge/3

nasty_car_or_cdr/1       boyer         ugliness/3

nontrivial/1             boyer         nasty_car_or_cdr/1

open_eval/2              boyer         expand/4

open_fn/2                boyer         expand/4

pickindvars/3            boyer         prove1/2

prove/1                  boyer         prove1/2 prove_base/2 prove_step/2 z/0 y/0
```

```
prove1/2                boyer         prove/1

prove_base/2            boyer         prove_by_induction/2

prove_by_induction/2    boyer         prove1/2

prove_step/2            boyer         prove_by_induction/2

rewrite/3               boyer         symbol_eval/3 rewrite/3

select/3                utility       merge/3

subst/3                 utility       prove_base/2 prove_step/2 fertilise/3

symbol_eval/3           boyer         prove/1      symbol_eval/3       rewrite/3
                                      prove_step/2

ugliness/3              boyer         expand/4 find_ugly/3

  itef/1                utility       prove1/2 prove_base/2 prove_step/2

writef/2                utility       prove/1

y/0                     boyer

z/0                     boyer
```

```
/* BOYER.

                                                  Leon Sterling
                                                  Updated: 24 July 81

A simple Boyer-Moore theorem prover as in
                Chap. 11 The Productive Use of Failure
                in 'Artificial Mathematicians'.

        The code written here is a simplified version of the algorithm
described in a paper of J Moore
'Computational Logic: Structure Sharing and Proof of Program Properties',
Xerox report CSL 75-2, which appeared also as Dept. of Computational Logic
memo no. 68, and the second part of Moore's Ph.D. thesis.
This is a simplified version of the theorem-prover described in the book
'A Computational Logic' by Boyer and Moore.

        Variable and procedure names have been chosen to be the same
as much as possible, and the mini-projects will be aided by the description
in the paper.

*/

%% PROOF STRATEGY %%


                           % To prove a theorem use the following algorithm
                           %   (on p.   of 'Artificial Mathematicians') :
                           % 1.-2. Try symbolic evaluation, recording the
                           %       reasons for failure in the list Analysis.
                           % 3.-4. If unsuccessful, try proof by induction
                           %   using the previously generated failure list
                           %   to suggest the induction scheme.
                           % 5. Finally try generalising the theorem if the
                           %    induction was unsuccessful.

prove(A) :-
        writef('\nTrying to prove %t\n',[A]),
        symbol_eval(A,B,Analysis),                % Try symbolic evaluation
        prove1(B,Analysis).


prove1(tt,_) :-                           %  Symbolic evaluation was successful
        !,
        writef('Expression evaluated to tt\n').

prove1(A,Analysis) :-
        pickindvars(A,Analysis,Var),              % find induction candidate
        prove_by_induction(A,Var).

prove1(A,Analysis) :-
        generalise(A,New),
        !,
        prove(New=tt).


%% SYMBOLIC EVALUATION %%
```

generalize not defined
eval multi-defined
add-bounds not called
not called

35

```
                           % Symbolic evaluation is performed by the procedure
                           %  symbol_eval(A,New,Analysis) which evaluates
                           %  expression A into expression New producing
                           %  failure bag Analysis.
                           % Primitive pure LISP functions, i.e. car,cdr,cond
                           %  and equal are handled by an evaluator if they
                           %  can be simplified. Otherwise function arguments
                           %  are symbolically evaluated bottom-up,
                           %  bottoming out on atomic expressions.  Function
                           %  definitions are expanded according to the criteria
                           %  described in the section of code
                           %       %% EXPANSION OF FUNCTION DEFINITIONS %%
                           %

symbol_eval(tt,tt,[]) :- !.      % Finished if expression evaluates to tt

symbol_eval([],[],[]) :- !.

symbol_eval(A,B,Analysis)  :-
        eval(A,A1),                          % Evaluate primitive expressions
        !,
        symbol_eval(A1,B,Analysis).

symbol_eval(A,B,Analysis) :-                        % Recursively rewrite terms
        A=..[Pred|Args],                            %  bottom-up.
        rewrite(Args,Args1,Anal),
        A1=..[Pred|Args1],
        expand(Pred,A1,A2,Fault),                   % Expand recursively defined
        merge(Fault,Anal,Analysis),                 % non-primitive functions
        def_eval(A2,B).


rewrite([],[],[]) :- !.

rewrite(X,X,[]) :- atomic(X), !.

rewrite([H|T],[H1|T1],Analysis) :-
        symbol_eval(H,H1,Fault),
        rewrite(T,T1,Desc),
        merge(Fault,Desc,Analysis).


                           % Evaluator

eval(car([]),[]) :- !.
eval(cdr([]),[]) :- !.
eval(car(cons(H,T)),H) :- !.
eval(cdr(cons(H,T)),T) :- !.
eval(cond([],U,V),V) :- !.
eval(cond(cons(X,Y),U,V),U) :- !.
eval(equal(X,X),tt) :- !.



%% EXPANSION OF FUNCTION DEFINITIONS %%

                           % Functions that can be expanded according to
                           %  their function definition are contained in an
                           %  open_fn predicate.  In rewriting expressions
                           %  functions are expanded where possible
```

```
                            %  using the predicate open_eval unless
                            %  ugly expressions are found. In this case the
                            %  fault description is returned as described
                            %  in section 3.2 of Moore's paper.
                            %

expand(Pred,Clause,Clause,Fault) :-
        open_fn(Clause,Newclause),
        ugliness(Pred,Newclause,Bomb),
        Bomb\==[],
        !,
        make_fault(Bomb,Fault).

expand(Pred,Clause,Newclause,[]) :-
        open_eval(Clause,Newclause).

expand(Pred,Clause,Clause,[]).

open_fn(append(X,Y),cond(X,cons(car(X),append(cdr(X),Y)),Y)) :- !.

open_eval(append(X,Y),Clause) :-
        def_eval(car(X),C1),
        def_eval(cdr(X),C2),
        def_eval(cond(X,cons(C1,append(C2,Y)),Y),Clause).

def_eval(X,Y) :- eval(X,Y), !.
def_eval(X,X).

                            % Make up a fault description from the faults
                            %  returned when trying to expand a recursively
                            %  defined function.

make_fault([],[]) :- !.

make_fault([fault(B,F)|Bombs],[fault(B,F)|Faults]) :-
        make_fault(Bombs,Faults).

make_fault([bomb(X)],[fault(bomb(X),fail([]))]).

make_fault([fail(X)],[fault(bomb([]),fail(X))]).

make_fault([bomb(X),fail(Y)],[fault(bomb(X),fail(Y))]).

make_fault([fail(Y),bomb(X)],[fault(bomb(X),fail(Y))]).


                            % Is expression ugly?

ugliness(Pred,X,[]) :- atomic(X), !.

ugliness(Pred,X,Analysis) :-
        nasty_car_or_cdr(X),
        !,
        analyse(Pred,X,Analysis).

ugliness(_,X,Analysis) :-
        X=..[Pred|Args],
        find_ugly(Pred,Args,Analysis).

find_ugly(_,[],[]) :- !.
```

```
find_ugly(Pred,[H|T],Analysis) :-
        ugliness(Pred,H,H1),
        find_ugly(Pred,T,T1),
        append(H1,T1,Analysis),
        !.

nasty_car_or_cdr(car(X)) :- nontrivial(X), !.
nasty_car_or_cdr(cdr(X)) :- nontrivial(X), !.

add_bomb(X,[],X) :- !.
add_bomb(Y,X,Z) :- append(Y,[X],Z).

analyse(Pred,X,[bomb(X)]) :- loop_threat(Pred,X), !.
analyse(Pred,X,[fail(X)]).

loop_threat(append,car(X)).
loop_threat(append,cdr(X)).




nontrivial(X) :- memberchk(X,[cons(U,V),[]]), !, fail.
nontrivial(X).


%% PROOF BY INDUCTION %%

                        % Find induction candidate
                        %  A simple majority vote is used to decide
                        %  which list to induct on.
                        %  This is calculated by max.

pickindvars(_,Bag,Var) :-
        max(Bag,[],Term,0,N),
        Term = fault(bomb(cdr(Var)),fail(_)).


max([],A,A,N,N) :- !.

max([pair(Pred,M)|Rest],Current,Ans,N,Num_ans) :-
        M > N,
        !,
        max(Rest,Pred,Ans,M,Num_ans).

max([_|Rest],Current,Ans,N,Num_ans) :-
        max(Rest,Current,Ans,N,Num_ans).

                        % Successively prove the base and step cases

prove_by_induction(A,Literal) :-
        prove_base(A,Literal),
        prove_step(A,Literal).

                        % To prove the base case, substitute the nil list
                        %  for the induction variable, and try to prove
                        %  the resultant clause

prove_base(A,Literal) :-
        writef('\n Base case'),
```

```
        subst(Literal=[],A,New),
        prove(New),
        !.

                        % To prove the step case, substitute the appropriate
                        %  cons expression into the clause, symbolically
                        %  evaluate as much as possible, then fertilise
                        %  to prove the expression

prove_step(A,Literal) :-
        writef('\nStep case'),
        subst(Literal=cons(a1,Literal),A,New),
        symbol_eval(New,Clause,_),
        fertilise(A,Clause,Newclause),
        prove(Newclause).

fertilise(equal(X,Y),Clause,New) :-     subst(X=Y,Clause,New).


%% PROOF BY GENERALISATION %%

                                % Code to be written


% test cases
z :- prove(equal(append([],x),x)).
y :- prove(equal(append(a,append(b,c)),append(append(a,b),c))).


/* Mini-projects

1. Add definitions of functions like reverse and copy to enable
        the theorem-prover to work on other examples. Explain
        how the theorem-prover might be modified to overcome
        any small problems that might arise.

2. Use more sophisticated criteria to choose the induction variable,
        or more generally the induction schema to be used.
        This will probably involve changing the way the failures
        are returned in the variable Analysis.

3. Write a more powerful version of fertilise.

4. Write code to perform generalise.

*/


                        % merge merges two bags

merge([],Bag,Bag) :- !.

merge(Bag,Var,Hack) :- var(Var), !, merge(Bag,[variable],Hack).

merge([pair(Ugly,K)|T],Bag,Newbag) :-
        select(pair(Ugly,N),Bag,Rest),
        !,
        M is N + K,
        merge(T,[pair(Ugly,M)|Rest],Newbag).
```

```
merge([pair(Ugly,N)|T],Bas,Newbas) :-
        !,
        merge(T,[pair(Ugly,N)|Bas],Newbas).

merge([H|T],Bas,Newbas) :-
        select(pair(H,N),Bas,Rest),
        !,
        M is N + 1,
        merge(T,[pair(H,M)|Rest],Newbas).

merge([H|T],Bas,Newbas) :-
        merge(T,[pair(H,1)|Bas],Newbas).
```

```
/*      Dska:Taut.[400,422]                        8 July 1981.

        A simple tautology checker based on the one by Boyer & Moore.
    The basic idea is to take a propositional formula formed from the
    constants '0'=false, '1'=true, propositional variables represented
    by Prolog atoms, and the functors not/1, and/2, or/2, imp/2, if/3.
    This is then converted to an equivalent formula using 'if' alone,
    where the test is a single propositional variable.  Then the cases
    of the test variables are examined.

*/

:- op(100,  fy, not).
:- op(200, xfy, and).
:- op(300, xfy,  or).
:- op(400, xfx, imp).

rewrite(if(Test, Left, Right), Answer) :-
        rewrite(Test,   Test1),
        rewrite(Left,   Left1),
        rewrite(Right,  Right1),
        buildif(Test1, Left1, Right1, Answer).

rewrite(and(Left, Right), Answer) :-
        rewrite(Left,   Left1),
        rewrite(Right,  Right1),
        buildif(Left1, Right1, 0, Answer).

rewrite(or(Left, Right), Answer) :-
        rewrite(Left,   Left1),
        rewrite(Right,  Right1),
        buildif(Left1, 1, Right1, Answer).

rewrite(imp(Left, Right), Answer) :-
        rewrite(Left,   Left1),
        rewrite(Right,  Right1),
        buildif(Left1, Right1, 1, Answer).

rewrite(not(Left), Answer) :-
        rewrite(Left,   Left1),
        buildif(Left1, 0, 1, Answer).

rewrite(Answer, Answer) :-
        atomic(Answer).



buildif(T, 1, 0, T).

buildif(T, L, L, L).

buildif(1, L, R, L).

buildif(0, L, R, R).

buildif(T, L, R, if(T, L, R)) :- atom(T).

buildif(if(T, TL, TR), L, R, if(T, AL, AR)) :-
        buildif(TL, L, R, AL),
        buildif(TR, L, R, AR).
```

```
/*      An association list is a list of variable=value terms, where each
        variable is a propositional variable, and the value is 0 or 1.

*/

lookup(Atom, [Atom=Value|_], Value) :- !.
lookup(Atom, [_|Rest],       Value) :- !, lookup(Atom, Rest, Value).
lookup(Atom, [],                 2).

check(if(Test, Left, Right), Alist, Value) :-
        lookup(Test, Alist, Selector), !,
        cases(Selector, Test, Left, Right, Alist, Value).
check(1,                     Alist,     1).
check(0,                     Alist,     0).
check(Atom,                  Alist, Value) :-
        atom(Atom),
        lookup(Atom, Alist, Value).

/* check(Expr, Alist, 0) => Expr can only be false under Alist,
   check(Expr, Alist, 1) => Expr can only be true  under Alist,
   check(Expr, Alist, 2) => Expr may be true or false.
*/

cases(0, Test, Left, Right, Alist, Value) :-
        check(Right, Alist, Value).

cases(1, Test, Left, Right, Alist, Value) :-
        check(Left,  Alist, Value).

cases(2, Test, Left, Right, Alist, Value) :-
        check(Left,  [Test=1|Alist], VL),
        check(Right, [Test=0|Alist], VR),
        combine(VL, VR, Value).


combine(X, X, X).
combine(X, Y, 2) :- X \== Y.


/* The top level routine */

classify(Formula) :-
        rewrite(Formula, IfTree),
        check(IfTree, [], Value),
        describe(Value, Formula).

describe(2, Formula) :- write(Formula), write(' is contingent.').
describe(1, Formula) :- write(Formula), write(' is always true.').
describe(0, Formula) :- write(Formula), write(' is always false.').
```

```
/* RULES.

Production Rule System for AI1
Alan Bundy 17.9.81

Use with UTIL. Example rules etc on file SUBTRA and SUM.
*/


go :-                   % Top level goal
        apply_rule,     % Apply a production rule
        go.             % and recurse


apply_rule :-                           % To apply a rule
        rule(Name,Condition, Action),   % Find a production rule
        satisfied(Condition),           % Check that its condition list is satisfi
        refract(Name,Condition),        % Check that rule has not already been fir
        writef('Rule %t fired\n',[Name]),
        Action.                         % If so, run its action

satisfied(Cond1 & Cond2) :- !,                  % To satisfy two conditions
        satisfied(Cond1), satisfied(Cond2).     % satisfy one after another

satisfied(Condition) :-                                 % To check a condition
        short_memory(B1,B2,B3,B4,B5,B6),                % Get short term memory
        memberchk(Condition,[B1,B2,B3,B4,B5,B6]).       % Match condition

add(Item) :-                                            % To add item to memory
        retract(short_memory(B1,B2,B3,B4,B5,B6)),       % recover & delete memory
        assert(short_memory(Item,B1,B2,B3,B4,B5)),      % add new item & drop B6
        writef('%t remembered\n',[Item]).


refract(Name,Cond) :-                           % Refractoriness
        history(HistList),                      % Recall history
        memberchk(pair(Name,Cond),HistList),    % If we have been here before
        !, fail.                                % then fail

refract(Name,Cond) :-
        retract(history(HistList)),             % otherwise update history
        assert(history([pair(Name,Cond) | HistList])).

history([]).            % Initial setting for history
```

```
/* SUBTRA.

Production Rules for Subtraction by Decomposition
from O'Shea and Young WP42
Alan Bundy 22.9.81

Example for RULES, use with UTIL
*/

/* The Rules */

rule(fin,next_column, shift_left & take_diff & write_answer & abort).
rule(b2a,s_gtr_m, add(borrow)).
rule(b2c,s_eq_m, result(0) & add(next_column)).
rule(bs2,borrow, decrement).
rule(bs3,borrow, add_ten_to_m).
rule(cm,process_column, compare).
rule(ts,process_column, take_diff & add(next_column)).



/* The Actions */

shift_left :-
        retract(mark(X)),
        X1 is X+1,
        assert(mark(X1)).

take_diff :-
        mark(X),
        column(X,M,S),
        pos_diff(M,S,R),
        assert(answer(X,R)).

pos_diff(M,S,R) :-
        M>=S, !, R is M-S.

pos_diff(M,S,R) :-
        R is S-M.

result(R) :-
        mark(X),
        assert(answer(X,R)).

add_ten_to_m :-
        mark(X),
        retract(column(X,M,S)),
        M1 is M+10,
        assert(column(X1,M1,S)).

decrement :-
        mark(X), X1 is X+1,
        retract(column(X1,M,S)),
        M1 is M-1,
        assert(column(X1,M1,S)).

compare :-
        mark(X),
        column(X,M,S),
        compare1(M,S,Verdict),
```

```
        add(Verdict).

compare1(M,M,s_eq_m).
compare1(M,S,s_str_m) :- S>M.
compare1(M,S,m_str_s) :- M>S.


write_answer :-
        column(1,M1,S1), answer(1,R1),
        column(2,M2,S2), answer(2,R2),
        writef('%t %t\n%t %t\n---\n%t %t\n---\n\n',
                [M2,M1,S2,S1,R2,R1]).


/* Short Term Memory */
short_memory(process_column,nil,nil,nil,nil,nil).
```

```
/* SUM1.

Example sum for use with RULES and SUBTRA
Alan Bundy 23.9.81

*/

column(1,8,9).
column(2,4,1).
mark(1).
```

```
/* SUM2.

Example sum for use with RULES and SUBTRA
Alan Bundy 23.9.81

*/

column(1,9,8).
column(2,4,1).
mark(1).
```

```
/* SUM3.

Example sum for use with RULES and SUBTRA
Alan Bundy 23.9.81

*/

column(1,8,8).
column(2,4,1).
mark(1).
```

```
yes
| ?- so.
Rule cm fired
s_str_m remembered
Rule b2a fired
borrow remembered
Rule bs2 fired
Rule bs3 fired
Rule ts fired
next_column remembered
Rule fin fired
3 18
 1 9
 ---
 2 9
 ---

[ Execution aborted ]

| ?- restore(foo).

yes
| ?- [sum2].

sum2 consulted   16 words      0.01 sec.

yes
| ?- so.
Rule cm fired
m_str_s remembered
Rule ts fired
next_column remembered
Rule fin fired
4 9
 1 8
 ---
 3 1
 ---

[ Execution aborted ]

| ?- restore(foo).

yes
| ?- [sum3].

sum3 consulted   16 words      0.01 sec.

yes
| ?- so.
Rule cm fired
s_eq_m remembered
Rule b2c fired
next_column remembered
Rule fin fired
4 8
 1 8
 ---
 3 0
```

```
---

[ Execution aborted ]

| ?- core     60416  (31232 lo-seg + 29184 hi-seg)
heap     26112 =  23975 in use +   2137 free
global    1173 =     16 in use +   1157 free
local     1024 =     16 in use +   1008 free
trail      511 =      0 in use +    511 free
    0.01 sec. for 5 trail shifts
    1.25 sec. runtime
```

```
                             **********************************
                             * PROLOG CROSS REFERENCE LISTING *
                             **********************************

                                   MYCIN CROSS REFERENCE


  PREDICATE                  FILE          CALLED BY



answer_query/1           mycin          check_for_query/4

askfor/3                 mycin          sethypos/3

  eck_again_for_query/2 mycin          check_for_query/4

check_ans/5              mycin          askfor/3 check_ans/5

check_for_more/2         mycin          getdata/2

check_for_query/4        mycin          askfor/3

compare/4                mycin          merge/2 compare/4

consider/3               mycin          check_for_more/2

consult/0                mycin          start_session/0

current/2                mycin          getdata/2 transpred/1

data_class/3             mycin          gennew/2       getdescr/3       current/2
                                        check_for_more/2

   duce/3                mycin          sethypos/3

divide/3                 mycin          explain/1 divide/3

explain/1                mycin          report/2

f/1                      mycin          therapy_required/0 check_again_for_query/2

genheader/2              mycin          gennew/2

genmess/2                mycin          getdescr/3

gennew/2                 mycin          getdata/2

genno/2                  mycin          gennew/2 getans/3

get/4                    mycin          same/4
```

```
get_nearest/1           mycin         check_for_query/4

getans/3                mycin         getlist/3

getdata/2               mycin         consult/0 getdata/2 consider/3

getdescr/3              mycin         getdata/2

gethypos/3              mycin         getdata/2 get/4

getlist/3               mycin         getdescr/3 getlist/3

give_evidence/1         mycin         transpred/1

gram/4                  mycin         readans/2

intersect/3             utility

intersect/3             mycin         get/4 intersect/3 known/1

know/3                  undefined     gethypos/3 known/1

known/1                 mycin         divide/3

last_class/2            mycin         getdata/2

maxhyp/2                mycin         get/4 maxhyp/2 known/1

member/2                utility

member/2                mycin         check_ans/5 member/2 get_nearest/1

merge/2                 mycin         deduce/3 merge/2

min/2                   mycin         min/2 rule/6

number/2                undefined     genno/2 current/2

  mber/3                mycin         gram/4 number/3

parameter/2             mycin         check_ans/5

question/1              mycin         getans/3

question/2              mycin         askfor/3

readans/2               mycin         askfor/3 check_ans/5

report/2                mycin         check_for_query/4

repwrite/2              mycin         genheader/2 repwrite/2 getans/3

rule/6                  mycin         rule_check/4

rule_check/4            mycin         <user> deduce/3
```

```
ruleno/1                mycin

same/4                  mycin         rule/6

space/2                 mycin         gram/4 space/2 word/3 number/3

start_session/0         mycin

test/4                  mycin         check_again_for_query/2

therapy_required/0      mycin         start_session/0

translate/2             mycin         report/2 explain/1

transpred/1             mycin         writeprems/1 translate/2

union/3                 utility

  ion/3                 mycin         intersect/3

word/3                  mycin         gram/4 word/3

write_known/1           mycin         explain/1

write_list/1            mycin         check_ans/5 write_list/1

writeprems/1            mycin         write_known/1 writeprems/1 translate/2

writeval/1              mycin
```

```
FILE  mycin

%declarations%

:- public
                rule_check/4.

% imports:
%               know/3                      (from undefined)
%               number/2                    (from undefined)

%end%
```

```
%       Mycin(Prolog)   - a Prolog rational reconstruction.
%       Waterloo Prolog version by Peter Hammond 1980.
%       converted to Dec-10 Prolog by Richard O'Keefe 1981.

%       A consultation session is initiated by the goal start_session.

start_session :- therapy_required, !, consult.
start_session :- write('The patient needs no therapy.'), nl.

therapy_required :- f(getans(0, initiator, Ans)), Ans = yes.

consult :- getdata(0, PatientData), write(PatientData), nl.

%       The hierarchical nature of the structure of PatientData is reflected
%       in the recursive definition of getdata below.

getdata(Class4, Hypos) :-
        Class3 is Class4-1,
        current(Class3, Entity),
        last_class(Class4, Attribute), !,
        gethypos(Entity, Attribute, Hypos).
 -tdata(Class1, [Class1DataItem | OtherClass1Data]) :-
        Class1 < 4,
        gennew(Class1, Entity),
        getdescr(Class1, Entity, Descr),
        Class2 is Class1+1,
        getdata(Class2, Class2Data),
        Class1DataItem = w(Entity, Descr, Class2Data),
        check_for_more(Class1, OtherClass1Data).

%       gennew generates a new entity of the required class and also
%       prints a heading announcing the new entity.

gennew(Class, Entity) :-
        genno(Class, NewNo),
        data_class(Class, ClassName, Details),
        Entity = ClassName-NewNo,       % e.g. patient-1
        genheader(Class, Entity).

%       Each time a new entity is required genno generates a suitable
%       cardinal for the entity.

genno(Class, NewNo) :-
        retract(number(Class, OldNo)), !,
        NewNo is OldNo+1,
        assert(number(Class, NewNo)).
genno(Class, 1) :-
        \+ number(Class, _),
        assert(number(Class, 1)).

genheader(ClassNo, Entity) :-
        repwrite('  ', ClassNo), repwrite('- ', 8),
        write(Entity), repwrite(' -', 8), nl.

repwrite(Object, Times) :-
        Times > 0, Left is Times-1,
        write(Object), !,
        repwrite(Object, Left).
repwrite(Object, 0).
```

```
%       setdescr asks the user to supply information of a background nature
%       for a particular entity, and can also cause a message to be printed
%       announcing the name of the first entity in the next class in the hierarch

setdescr(Class, Entity, Descr) :-
        data_class(Class, Name, Details),
        setlist(Class, Details, Descr),
        genmess(Class, Entity).

setlist(Class, [Item | OtherItems], [Ans | OtherAns]) :-
        getans(Class, Item, Ans),
        setlist(Class, OtherItems, OtherAns).
setlist(Class, [], []).

%       current returns the current entity of a particular class.

current(Class, Entity) :-
        number(Class, No),      %       fails if none yet
        data_class(Class, ClassName, Details),
        Entity = ClassName-No.

%       getans asks the user for and reads the value of an item of background
        data.  Each question is preceded by a new question number.

getans(Class, Item, Ans) :-
        genno(question, Q),
        repwrite('  ', Class),
        display('('), display(Q), display(') '),
        question(Item), ttyflush,
        read(Ans), nl.

%       check_for_more asks the user if there is another entity of a
%       particular class to consider.

check_for_more(Class, OtherData) :-
        data_class(Class, ClassName, Details),
        display('Is there another '),
        display(ClassName), display('? '), ttyflush,
        read(Ans),
        consider(Ans, Class, OtherData).

consider(no,  Class, []).
consider(yes, Class, OtherData) :- getdata(Class, OtherData).

%       same causes evidence to be gathered which bears on a particular
%       value of a clinical parameter, and succeeds if the CF supporting
%       this value is greater than 200.

same(Entity, Attribute, ReqVal, CFmi) :-
        get(Entity, Attribute, ReqVal, CFmi), !,
        CFmi > 200.

%       get collects together the hypotheses relevant to determining the
%       value of a clinical parameter and finds the largest of the CF-s
%       supporting the possible values.

get(Entity, Attribute, ReqVal, CFmi) :-
        gethypos(Entity, Attribute, Hypos),
        intersect(Hypos, ReqVal, Intersect),
        maxhyp(Intersect, CFmi).
```

```
sethypos(Entity, Attribute, Hypos) :-
        (   know(Entity, Attribute, Hypos)
        ;   deduce(Entity, Attribute, Hypos)
        ;   askfor(Entity, Attribute, Hypos)
        ),
        Hypos \== [].

%       deduce collects all the evidence for the value of a parameter,
%       merges evidence for the same value into one hypothesis, and
%       stores the information obtained.

deduce(Entity, Attribute, Hypos) :-
        bagof(v(Val, Cf), rule_check(Entity, Attribute, Val, Cf), H),
        merge(H, Hypos),
        assert(know(Entity, Attribute, Hypos)).

%       rule_check investigates a rule and calculates the Cf of the
%       deduction when the rule succeeds.

rule_check(Entity, Attribute, Value, Cf) :-
        rule(RuleNo, Entity, Attribute, Value, C, Tally),
        Cf is Tally*C/1000.

merge([], []).
merge([v(unk,1000)], []).
merge([H | Rest], [H1 | Rest1]) :-
        compare(H, Rest, H1, Tser1),
        merge(Tser1, Rest1).

compare(R, [], R, []).
compare(v(Val, Cf1), [v(Val, Cf2) | U], R, W) :-
        Cf3 is (1000-Cf1)*Cf2/1000 + Cf1,
        compare(v(Val, Cf3), U, R, W).
compare(v(Val1, Cf1), [v(Val2, Cf2) | U], R, [v(Val2, Cf2) | W]) :-
        Val1 \== Val2,
        compare(v(Val1, Cf1), U, R, W).

intersect([v(Val, Cf) | H], [Val | Rest], [v(Val, Cf) | H1]) :- !,
        intersect(H, Rest, H1).
  tersect([v(Val, Cf) | H], [Alt | Rest], H1) :- !,
        intersect(H, [Alt], X),
        intersect([v(Val, Cf) | H], Rest, Y),
        union(X, Y, H1).
intersect([], List, []).
intersect(Hypos, [], []).

union([], Y, Y).
union(X, [], X).
union([R], Y, [R|Y]).

maxhyp([], 0).
maxhyp([v(Val, Cf) | H], CFmi) :-
        maxhyp(H, C1),
        (C1 > Cf, !, CFmi = C1; CFmi = Cf).

%       askfor causes the user to be asked for the value of a clinical
%       parameter.  The answer is read and checked to see if it is one
%       of the question's possible answers.
```

```
askfor(Entity, Attribute, [v(ActVal, Cf)]) :-
        question(Entity, Attribute),
        readans(Answer1, Cf1),
        check_for_query(Answer1, Cf1, A, C),
        check_ans(Attribute, A, C, ActVal, Cf),
        assert(know(Entity, Attribute, [v(ActVal, Cf)])).

check_ans(Attribute, A1, C1, A1, C1) :-
        parameter(Attribute, Expected),
        member(A1, Expected), !.
check_ans(Attribute, A1, C1, A, C) :-
        parameter(Attribute, Expected),
        display('Please enter one of the following:'), nl,
        write_list(Expected),
        readans(Answer2, Cf2),
        check_ans(Attribute, Answer2, Cf2, A, C).

question(Entity, Attribute) :-
        display('        Please enter the '), display(Attribute),
        display(' of '), write(Entity), display(': '), ttyflush.

%       readans is used to read the user's reply to a request for the value
        of a clinical parameter.  The value and its certainty factor are
%       both read.  The default Cf is 1000.

readans(Answer, Cf) :-
        read(Reply),
        name(Reply, Text),
        gram(Answer, Cf, Text, []).

        gram(Answer, Cf) --> space, word(A), {name(Answer, A)},
                             (    "(", space, number(C), ")", space,
                                 {name(Cf, C)} ; {Cf = 1000}).

                space --> [C], {C =< 32}, space | [].

                word([C|R]) --> [C], {C > 32, C =\= "("}, word(R).
                word(  [] ) --> space.

                number([C|R]) --> [C], {C >= "0", C =< "9"}, number(R).
                number(  [] ) --> space.

member(X, [X|_]).
member(X, [_|R]) :- member(X, R).


write_list([X|Y]) :-
        write(X), nl, write_list(Y).
write_list([]) :-
        nl.

min([X], X).
min([X|Y], Z) :-
        min(Y, U),
        ( X < U, !, Z=X ; Z=U ).

%       T H E   E X P L A N A T I O N   S Y S T E M

%       check_for_query is called after the user is asked to give a parameter
%       value.  If he said WHY or RULE the explanation system is entered.
```

```
check_for_query(Answer, Cf, A, C) :-
        (   Answer = why   ;   Answer = rule    ), !,
        answer_query(Answer),
        get_nearest(Rule),
        report(Answer, Rule),
        check_again_for_query(A, C).
check_for_query(Answer, Cf, Answer, Cf).

answer_query(why) :-            % he can't be serious!
        display('To determine the genus of the organism.'), nl.
answer_query(rule) :-
        display('The current rule is:'), nl.

check_again_for_query(A, C) :-
        f(readans(A1, C1)),
        test(A1, C1, A, C).

test(why, 1000, A, C) :- !, fail.
test(A, C, A, C) :- A \== why.

 t_nearest(rule(N, E, A, R, C, T)) :-
        ancestors(AllOfThem),
        member(rule(N, E, A, R, C, T), AllOfThem).

report(why,  Rule) :- explain(Rule).
report(rule, Rule) :- clause(Rule, Body), translate(Rule, Body).

%       explain prints the known parameter values in a rule, those still
%       to be determined, and the deduction which could result.

explain(Rule) :-
        clause(Rule, Body),
        divide(Body, KnownPrems, UnknownPrems),
        write_known(KnownPrems),
        translate(Head, UnknownPrems).

divide(','(A,B), ','(A, OtherKnown), Unknown) :-
        known(A),
        divide(B, OtherKnown, Unknown).
  vide(','(A,B), true, ','(A,B)) :-
        \+ known(A).
divide(A, true, A) :-
        \+ known(A).

write_known(true).
write_known(','(A,B)) :-
        display('       It is known that:'), nl,
        writeprems(','(A,B)),
        display('       therefore...'), nl.

writeprems(true).
writeprems(','(A,B)) :-
        transpred(A),
        writeprems(B).
writeprems(A) :-
        transpred(A).

%       translate gives a simple english translation of a rule.
```

```
translate(Head, Body) :-
        display('       If :'), nl,
        writeprems(Body),
        display('       Then :'), nl,
        transpred(Head).

transpred(min(M,N)).
transpred(same(E, A, R, C)) :-
        current(3, Entity),
        display('            the '), write(A),
        display(' of '), write(Entity),
        display(' is '), write(R), nl.  %  the genus of organism-1 is [staph]
transpred(rule(N, E, A, V< C, T)) :-
        current(3, Entity),
        display('            there is '),
        give_evidence(C),
        display(' evidence that the '), nl,
        display('            '), write(A),
        display(' of '), write(E),
        display(' is '), write(V), nl,
        display('                (rule '),
        write(N), display(')'), nl.

known(same(E,A,R,C)) :-
        know(E, A, Hypos),
        intersect(Hypos, R, I),
        maxhyp(I, CFmi),
        CFmi > 200.

ruleno(N) :-
        display('                (Rule '),
        display(N), display(')'), nl.

writeval([M]) :- write(M).

give_evidence(C) :-
        C > 800, display('strongly suggestive')
     ;  C > 400, display('suggestive')
     ;  C < 401, display('weakly suggestive')
        .
```

THE KNOWLEDGE BASE

```
%       data_class lets the user delcare the classes of data objects,
%       their names, and the background details required.

data_class(0, patient, [name, sex, age]).
data_class(1, infection, [inftype, infdate]).
data_class(2, culture, [cultsite, cultdate]).
data_class(3, organism, []).
last_class(4, genus).

genmess(0, Entity).
genmess(1, Entity) :-
        display('       The most recent culture associated with '),
        write(Entity),
        display(' will be referred to as:'), nl.
genmess(2, Entity) :-
        display('       The first significant organism from '),
        write(Entity),
```

```
        display(' will be referred to as:'), nl.
genmess(3, Entity).

%       question defines the questions which will elicit the values of
%       particular details from the user.

question(name) :-
        display('Patient''s name: ').
question(sex) :-
        display('Patient''s sex : ').
question(age) :-
        display('Patient''s age : ').

question(initiator) :-
        display('Have you been able to obtain positive cultures from a
site at which the patient has an infection? ').
question(inftype) :-
        display('What is the infection? ').
question(infdate) :-
        display('When did this infection first appear? ').
question(cultsite) :-
        display('What site did the specimen of this culture come from? ').
 uestion(cultdate) :-
         display('When was this culture obtained? ').

%       parameter records the possible values of a clinical parameter.

parameter(genus, [unk,strep,neiss,bact,staph,coryn]).
parameter(gramstain, [unk,pos,neg]).
parameter(morphology, [unk,rod,coccus]).
parameter(conformation, [unk,singles,longchains,shortchains]).
parameter(aerobicity, [unk,anaerobic,facul]).

%       T H E   * R U L E *   B A S E

rule(35,        Entity, genus,  bact,   600,    Tally) :-
        same(Entity, gramstain, [neg], Cf1),
        same(Entity, morphology, [rod], Cf2),
        same(Entity, aerobicity, [anaerobic], Cf3),
        min([Cf1,Cf2,Cf3], Tally).

 ule(9,         Entity, genus,  neiss,  800,    Tally) :-
        same(Entity, gramstain, [neg], Cf1),
        same(Entity, morphology, [coccus], Cf2),
        min([Cf1,Cf2], Tally).

rule(306,       Entity, genus,  staph,  700,    Tally) :-
        same(Entity, gramstain, [pos], Cf1),
        same(Entity, morphology, [coccus], Cf2),
        same(Entity, conformation, [singles], Cf3),
        min([Cf1,Cf2,Cf3], Tally).

rule(412,       Entity, genus,  strept, 950,    Tally) :-
        same(Entity, gramstain, [pos], Cf1),
        same(Entity, morphology, [coccus], Cf2),
        same(Entity, conformation, [longchains], Cf3),
        min([Cf1,Cf2,Cf3], Tally).

rule(413,       Entity, genus,  strept, 800,    Tally) :-
        same(Entity, gramstain, [pos], Cf1),
```

```
        same(Entity, morphology, [coccus], Cf2),
        same(Entity, conformation, [shortchains], Cf3),
        min([Cf1,Cf2,Cf3], Tally).

f(X) :- call(X), !.             %  awful name!


%       T H E   * E N D *
```